ANSWERS

1. **(c):** The relationship is $x:\frac{x^2}{2}$.
2. **(b):** The relationship is $7x:x$.
3. **(d):** The relationship is $x:x^2$
4. **(c):** The relationship is $x^2:(x+1)$
5. **(a):** The relationship is $2x+2:x$

ALPHABET ANALOGY

In this type of questions, two groups of alphabets related to each other in same way, are given. The candidate is required to find out this relationship and choose a group of alphabets which is related in the same way to a third group provided in the question.

Example 1: ABCD : OPQR : : WXYZ : ?

(a) EFGH **(b)** KLMN **(c)** QRST **(d)** STUV

Sol. Clearly, each letter of the first group is moved fourteen steps forward to obtain the corresponding letter of the second group. A similar relationship will exist between the third and fourth groups. Hence, the answer is (b).

EXERCISE

1. **AG : IO : : EK : ?**

   (a) LR **(b)** MS **(c)** PV **(d)** SY

2. **ACE : FHJ : : OQS : ?**

   (a) PRT **(b)** RTU **(c)** TVX **(d)** UWY

3. **ACEG : DFHJ : : QSUW: ?**

   (a) KMNP **(b)** MNPR **(c)** TQST **(d)** TVXZ

4. **kcaC : Cack : : QSUW : ?**

   (a) EmgF **(b)** EgmX **(c)** WUSQ **(d)** GmeF

5. **Dda : Add : : Rrb : ?**

   (a) BBr **(b)** Brr **(c)** RRR **(d)** DDA

ANSWERS

1. **(b) :** Each letter of the first group is moved eight steps forward to obtain the corresponding letter of the second group.
2. **(c) :** Each letter of the first group is moved five steps forward to obtain the corresponding letter of the second group.
3. **(d) :** Each letter of the first group is moved three steps forward to obtain the corresponding letter of the second group.
4. **(c) :** The order of the letters of the first group is reversed to obtain the second group.
5. **(b) :** The order of the letters of the first group is reversed .

2- CLASSIFICATION

'Classification' means 'to assort the items of a given group on the basis of certain common qualities they possess and then spot the stranger out'. In this test, you are given a group of certain items, out of which all except one are similar to one another in some manner. The candidate is required to choose this one item which does not fit into the given group.

Choosing the Odd Word

Example 1: *Choose the word which is least like the other words in the group.*

(a) Curd (b) Butter (c) Oil (d) Cheese

Sol. Here, all except *Oil* are products obtained from milk. Hence, the answer is **(c)**

EXERCISE

Question No 1 directions: *In each of the following questions, five words have been given, out of which four are alike in some manner and the fifth one is different. Choose out the odd one.*

1. **(a)** Kiwi **(b)** Eagle **(c)** Emu **(d)** Penguin **(e)** Ostrich

2. **(a)** Lake **(b)** Lagoon **(c)** River **(d)** Pool **(e)** Pond

3. **(a)** Arrow **(b)** Axe **(c)** Knife **(d)** Dagger **(e)** Sword

4. **(a)** Sun **(b)** Moon **(c)** Star **(d)** Mars **(e)** Universe

5. **(a)** House **(b)** Cottage **(c)** School **(d)** Palace **(e)** Hut

ANSWERS

1. **(b) :** All except *Eagle* are flightless birds.

2. **(c) :** All except *River* contain stagnant water.

3. **(a) :** All except *Arrow* are used while holding in hand.

4. *(e)* **:** All except *Universe* form a part of the universe.

5. **(c) :** All except *School* are dwelling places.

Question No 2 directions: *In each of the following questions, five words have been given, out of which four are alike in some manner and the fifth one is different. Choose out the odd one.*

1. **(a)** Lima **(b)** Algiers **(c)** New York **(d)** Tokyo **(e)** Beijing

2. **(a)** Potassium **(b)** Silicon **(c)** Zirconium **(d)** Gallium **(e)** Germanium

3. **(a)** Tortoise **(b)** Duck **(c)** Snake **(d)** Whale **(e)** Crow

4. **(a)** Feathers **(b)** Tentacles **(c)** Fins **(d)** Pseudopodia **(e)** Flagella

5. **(a)** Sleet **(b)** Fog **(c)** Rain **(d)** Vapour **(e)** Mist

ANSWERS

1. **(c) :** All except *New York* are capital cities.

2. **(a) :** All except *Potassium* are metals used in semiconductor devices.

3. **(d) :** All except *Whale* lay eggs.

4. **(a) :** All except *Feathers* are organs for movement in different organisms.
5. **(d) :** All except *Vapour* are different forms of precipitation.

CHOOSING THE ODD PAIR OF WORDS

In this type of questions, certain pairs of words are given out of which the words in all the pairs except one, bear a certain common relationship. The candidate is required to decipher this relationship and choose the pair in which the words are differently related, as the answer.

Examples: **Directions:** ***Choose the Odd Pair Of words.***

(a) Blacksmith : Anvil
(b) Carpenter : Saw
(c) Barber : Scissor
(d) Goldsmith : Ornaments

Sol. Clearly, the answer is **(d)**. In all other pairs, second is the tool used by the first.

EXERCISE

Directions: ***In each of the following questions, five pairs of words are given out of which the words in four pairs bear a certain common relationship. Choose the pair in which the words are differently related.***

1. (a) Mason : Wall (b) Cobbler : Shoe
(c) Farmer : Crop (d) Chef : Cook
2. (a) Bottle : Wine (b) Cup : Tea
(c) Pitcher : Water (d) Ball : Bat
3. (a) Stallion : Colt (b) Horse : Mare
(c) Dog : Bitch (d) Drake : Duck
4. (a) Ornithology : Birds (b) Mycology : Fungi
(c) Biology : Botany (d) Psychology : Algae
5. (a) Daring : Timid (b) Beautiful : Pretty
(c) Clear : Vague (d) Youth : Adult

ANSWERS

1. **(d) :** In all other pairs, second is prepared by the first.
2. **(d) :** In all other pairs, first is used to hold the second.
3. **(a) :** In all other pairs, second is the female of the first.
4. **(c) :** In all other pairs, first is the study of second.
5. **(b) :** In all other pairs, the two words are antonyms of each other.

3-SERIES COMPLETION

This topic deals with questions in which series of numbers or alphabetical letters are given, which are generally called as terms of the series. These terms follow a certain pattern throughout. The candidate is required to recognize this pattern and either complete the given series with the most suitable alternative or find the wrong term in the series.

Number Series

Example 1: Which number would replace question mark in the series 7, 12, 19, ?, 39.

(a) 29 **(b)** 28 **(c)** 26 **(d)** 24

Sol. Clearly, the given sequence follows the pattern: 5, 7, 9 … i.e. $7 + 5 = 12 + 7 = 19, \ldots$

$\therefore$ Missing number $= 19 + 9 = 28$. Hence, the answer is **(b)**

Example 2: Which is the number that comes next in the sequence : 0, 6, 24, 60, 120, 210 ?

(a) 240 **(b)** 290 **(c)** 336 **(d)** 504

Sol. **Clearly, the given series is 1^3-1, 2^3-2, 3^3-3, 4^3-4, 5^3-5, 6^3-6…**

$\therefore$ Next number $= 7^3 - 7 = 343 - 7 = 336$. Hence, the answer is **(c)**.

Example 3: Which is the number that comes next in the following sequence?4, 6, 12, 14, 28, 30,..

(a) 32 **(b)** 60 **(c)** 62 **(d)** 64

Sol. The given sequence is a combination of two series : I. 4, 12, 28, (…..) and II. 6, 14, 30.

Now, the pattern followed in each of the above two series is : + 8, + 16, + 32, … .

So, missing number $= (28 + 32) = 60$. Hence, the answer if **(b)**.

Example 4: Find out the missing number in the following sequence : 1, 3, 3, 6, 7, 9, ?, 12, 21.

(a) 10 **(b)** 11 **(c)** 12 **(d)** 13

Sol. Clearly, the given sequence is a combination of two series : I. 1, 3, 7, ?, 21 and II. 3, 6, 9, 12

The pattern followed in I is + 2, + 4, … ; and the pattern followed in II is + 3.

Thus, missing number $= 7 + 6 = 13$. Hence, the answer is **(d)**.

Example 5: Which fraction comes next in the sequence $\frac{1}{2}, \frac{3}{4}, \frac{5}{8}, \frac{7}{16}, ?$

(a) $\frac{9}{32}$ **(b)** $\frac{10}{17}$ **(c)** $\frac{11}{34}$ **(d)** $\frac{12}{35}$

Sol. Clearly, the numerators of the fractions in the given sequence form the series 1, 3, 5, 7, in which each term is obtained by adding 2 to the previous term. The denominators of the fractions form the series 2, 4, 8, 16, i.e. $2^1, 2^2, 2^3, 2^4$. So, the numerator of the next fractions will be (7 +2), i.e. 9 and the denominator will be 2^5, i.e. 32. The next term is $\frac{9}{32}$. Hence, the answer is **(a)**.

EXERCISE

Directions: *In each of the following questions, a number series is given with one term missing. Choose the correct alternative that will continue the same pattern and fill in the blank spaces.*

1. 1, 4, 9, 16, 25, (.....)

(a) 35 (b) 36 (c) 48 (d) 49

2. 20, 19, 17, (.....), 10, 5

(a) 12 (b) 13 (c) 14 (d) 15

3. 2, 3, 5, 7, 11, (.....), 17

(a) 12 (b) 13 (c) 14 (d) 15

4. 6, 11, 21, 36, 56, (.....)

(a) 42 (b) 51 (c) 81 (d) 91

5. 1, 6, 13, 22, 33, (.....)

(a) 44 (b) 45 (c) 46 (d) 47

ANSWERS

1. (b): The numbers are $1^2, 2^2, 3^2, 4^2, 5^2$. ∴ Missing number = $6^2 = 36$.
2. (c): The pattern is $-1, -2, \ldots$ ∴ Missing number = $17 - 3 = 14$.
3. (b): Clearly, the given series consists of prime numbers starting from 2.
4. (c): The pattern is $+5, +10, +15, +20, \ldots$ ∴ Missing number = $56 + 25 = 81$.
5. (c): The pattern is $+5, +7, +9, +11 \ldots$ ∴ Missing number = $33 + 13 = 46$.

ALPHABET SERIES

Example 1: What terms will fill the blank spaces ?

Z, X, V, T, R, (.....), (.....)

(a) O, K (b) N, M (c) K, S (d) P, N

Sol. Clearly, the given series consists of alternate letters in a reverse order. So, the missing terms would be P and N. Hence, the answer is (d).

EXERCISE

Directions: *In each of the following questions, various terms of a letter series are given with one term missing as shown by (?). Choose the missing term out of the given alternatives.*

1. U, O, I, ?, A

(a) E (b) C (c) S (d) G

2. Y, W, U, S, Q, ?, ?

(a) N, J (b) M, L (c) J, R (d) O, M

3. A, B, D, G, ?

(a) M (b) L (c) K (d) H

4. Z, U, Q, ?, L
(a) I (b) K (c) M (d) N

5. A, C, F, H, ?, M
(a) L (b) K (c) J (d) I

ANSWERS

1. **(a):** The series consists of vowels A, E, I, O, U written in a reverse order.
2. **(*d*):** The series consists of alternate letters in reverse order.
3. **(c):** The first, second, third, letters of the series are respectively moved one, two, three, steps backward to obtain the successive terms.
4. **(d):** The first, second, third, letters of the series are respectively moved five, four, three, steps forward to obtain the successive terms.
5. **(b):** The letters are alternately moved two and three steps forward to obtain the successive terms.

LETTER SERIES

This type of questions usually consist of a series of small letters which follow a certain pattern. However, some letters are missing from the series. These missing letters are then given in a proper sequence as one of the alternatives. The candidate is required to choose this alternative as the answer.

EXERCISE

1. **- - aba - - ba - ab**
(a) abbba (b) abbab (c) baabb (d) bbaba

2. **ab — — — b — bbaa —**
(a) abaab (b) abbab (c) baaab (d) babba

3. **— baa — aab — a — a**
(a) aabb (b) aaba (c) abab (d) baab

4. **ba—- bb— aba ——**
(a) ababb (b) baaab (c) bbaba (d) babbb

5. **aa — ab — — aaa — a**
(a) aaab (b) aabb (c) abab (d) baaa

ANSWERS

1. **(b):** The series is ab/ab/ab/ab/ab/ab. Thus, the pattern ab is repeated.
2. **(c):** The series is abb/aab/abb/aab. Thus, the patter abb, aab is repeated
3. **(c):** The series is aba/aba/aba/aba. Thus, the pattern aba is repeated.

4. **(d):** The series is bababb/bababb. Thus, the pattern bababb is repeated.
5. **(a):** The series is aaaaba/aaaaba. Thus, the pattern aaaaba is repeated.

4-CODING-DECODING

A CODE is a 'system of signals'. Therefore, **Coding** is a method of transmitting a message between the sender and the receiver without a third person knowing it. The Coding and Decoding Test is set up to judge the candidate's ability to decipher the rule that codes a particular word/message and break the code to decipher the message.

LETTER CODING

In these questions, the real alphabets in a word are replaced by certain other alphabets according to a specific rule to form its code. The candidate is required to detect the common rule and answer the questions accordingly.

Example 1. If in a certain language MYSTIFY is coded as NZTUJGZ, how is NEMESIS coded in that language?

(a) MDLHRDR (b) OFNFTJT (c) ODNHTDR (d) PGOKUGU

Sol. Clearly, each letter in the word MYSTIFY is moved one step forward to obtain the corresponding letter of the code.

So in NEMESIS, N will be coded as O, E as F, M as N and so on. Thus, the code becomes OFNFTJT. Hence, the answer is **(b)**.

EXERCISE

1. **If in a code, ALTERED is written as ZOGVIVW, then in the same code, RELATED would be written as**

(a) IVOZGVW (b) IVOZGWV (c) IVOGZVW (d) VIOZGVW

2. **In a certain code, TRIPPLE is written as SQHOOKD. How is DISPOSE written in that code?**

(a) CHRONRD (b) DSOESPI (c) ESJTPTF (d) ESOPSID

3. **If in a code language, COULD is written as BNTKC and MARGIN is written as LZQFHM, how will MOULDING be written in that code?**

(a) CHMFINTK (b) LNKTCHMF (c) LNTKCHMF (d) NITKHCMF

4. **In a certain code, MONKEY is written as XDJMNL. How is TIGER written in that code?**

(a) QDFHS (b) SDFHS (c) SHFDQ (d) UJHFS

5. **If FRAGRANCE is written as SBHSBODFG, how can IMPOSING be written?**

(a) NQPTJHOJ (b) NQPTJOHI (c) NQTPJOHJ (d) NQPTJOHJ

ANSWERS

1. (a) : Each letter of the word ALTERED is replaced by the letter which occupies the same position from the other end of the English alphabet, to obtain the code. Thus, A, the first

letter of the alphabet, is replaced by Z, the last letter.

2. (a) : Each letter in the word is moved one step backward to obtain the corresponding letter of the code.

3. (c) : Each letter in the word is moved one step backward to obtain the corresponding letter of the code.

4. (a) : The letters of the word are written in a reverse order and then each letter is moved one step backward to obtain the code.

5. (d) : Each letter in the word is moved one step forward and the first letter of the group so obtained is put at the end, to obtain the code.

NUMBER CODING

In these questions, either numerical code values are assigned to a word or alphabetical code letters are assigned to the numbers. The candidate is required to analyse the code as per the directions.

Example 1. If in a certain language A is coded as 1, b is coded as 2, and so on, how is BIDDIC coded in that code?

(a) 294493 (b) 284563 (c) 375582 (d) 394492

Sol. As given the letters are coded as:

A	B	C	D	E	F	G	H	I
1	2	3	4	5	6	7	8	9

So, in BIDDIC, B is coded as 2, I as 9, D as 4, and C as 3. Thus, BIDDIC is coded as 294493. Hence, the answer is (a).

EXERCISE

1. **If D = 4 and COVER = 63, then BASIS =?**

   (a) 49 (b) 50 (c) 54 (d) 55

2. **In a certain code, RIPPLE is written as 613382 and LIFE is written as 8192. How is PILLER written in that code?**

   (a) 318826 (b) 318286 (c) 618826 (d) 338816

3. **If ROSE is coded as 6821, CHAIR is coded as 73456 and PREACH is coded as 961473, what will be the code for SEARCH?**

   (a) 246173 (b) 214673 (c) 214763 (d) 216473

4. **If in a certain code, TWENTY is written as 863985 and ELEVEN is written as 323039, how is TWELVE written in that code?**

   (a) 863203 (b) 863584 (c) 863903 (d) 863063

5. **If PAINT is coded as 74128 and EXCEL is coded as 93596, then how would you encode ACCEPT?**

   (a) 455978 (b) 547978 (c) 554978 (d) 735961

ANSWERS

(b) **1. (b):** ... Clearly, in the given code, A = 1, B =2, C = 3, ... so that COVER = 3 + 15 + 22 + 5 + 18 = 63. Now, in BASIS, = 2 + 1 + 19 + 9 + 19 = 50. Hence, the answer is (b).

(c) **2. (a):** The alphabets are coded as shown :

R	I	P	L	E	F
6	1	3	8	2	9

(d) So, in PILLER, P is coded as 3, I as 1, L as 8, E as 2 and R as 6. Thus, the code for PILLER is 318826.

(e) **3. (b):** The alphabets are coded as shown :

R	O	S	E	C	H	A	I	P
6	8	2	1	7	3	4	5	9

(f) So, in SERACH, S is coded as 2, E as 1, A as 4, R as 6, C as 7, H as 3. Thus, the code for SEARCH is 214673.

(g) **4. (a):** The alphabets are coded as shown :

T	W	E	N	Y	L	V
8	6	3	9	5	2	0

(h) So, in TWELVE, T is coded as 8, W as 6, E as 3, L as 2, V as 0. Thus, the code for TWELVE in 863203.

(i) 5. (a): Clearly, in the given code, the alphabets are coded as follows :

P	A	I	N	T	E	X	C	L
7	4	1	2	8	9	3	5	6

(j) So, in ACCEPT, A is coded as 4, C as 5, E as 9, P as 7 and T and T as 8. Hence, the correct code is 455978 and therefore, the answer is **(a)**.

SUBSTITUTION

In this type of questions, some particular objects are assigned code names. Then a question is asked that is to be answered in the code language.

Example 1. If cook is called *butler*, *butler* is called *manager*, *manager* is called *teacher*, *teacher* is called *clerk* and *clerk* is called *principal*, who will teach in a class?

(a) Cook **(b)** Butler **(c)** Manager **(d)** Clerk

Sol. Clearly, a 'teacher' teaches in a class and as given 'teacher' is called 'clerk'. So, a 'clerk' will teach in the class. Hence, the answer is **(d)**.

EXERCISE

1. If *white* is called *blue, blue* is called *red, red* is called *yellow, yellow* is called *green, green* is called *black, black* is called *violet* and *violet* is called orange, what would be the colour of human blood?

   (a) Red (b) Green (c) Yellow (d) Violet

2. If room is called *bed, bed* is called *window, window* is called *flower* and *flower* is called *cooler,* on what would a man sleep?

   (a) Window (b) Bed (c) Flower (d) Cooler

3. If *orange* is called *butter, butter* is called *soap, soap* is called *ink, ink* is called *honey* and *honey* is called *orange,* which of the following is used for washing clothes?

   (a) Honey (b) Butter (c) Orange (d) Ink

4. If *sand* is called *air, air* is called *plateau, plateau* is called *well, well* is called *island* and *island* is called *sky,* then from where will a woman draw water?

   (a) Well (b) Island (c) Sky (d) Air

5. If *bangle* is called *cassette, cassette* is called *table, table* is called *game* and *game* is called *cupboard,* then which is played in the tape recorder?

   (a) Bangle (b) Cassette (c) Table (d) Cupboard

ANSWERS

1. **(c)** : The colour of the human blood is 'red' and as given, 'red' is called 'yellow'. So, the colour of human blood is 'yellow'.
2. **(a)** : A man sleeps on a 'bed' and as given, 'bed' is called 'window'. So, the man will sleep on the 'window'.
3. **(d)** : Clearly, 'soap' is used for washing the clothes. But, 'soap' is called 'ink'. So, 'ink' is used for washing the clothes.
4. **(b)** : A woman shall draw water from a 'well' but a 'well' is called 'island'. So, the woman will draw water from an 'island'.
5. **(c)** : Clearly, a 'cassette' is played in the tape-recorder. But a 'cassette' is called 'table'. So, a table will be played in the tape-recorder.

5-BLOOD RELATIONS

In these tests, the success of a candidate depends upon the knowledge of the blood relations, some of which are summarized below to help solve these tests.

Mother's or father's son	Brother
Mother's or father's daughter	Sister
Mother's or father's brother	Uncle
Mother's or father's sister	Aunt
Mother's or father's father	Grandfather

Mother's or father's mother	Grandmother
Sons's wife	Daughter-in-Law
Daughter's husband	Son-in-law
Husband's or wife's sister	Sister-in-Law
Husband's or wife's Brother	Brother-in-Law
Brother's son	Nephew
Brother's daughter	Niece
Uncle's or aunt's son or daughter	Cousin
Sister's husband	Brother-in-Law
Grandson's or Granddaughter's daughter	Great granddaughter

In this type of questions, a round-about description is given in the form of certain small relationships and direct relationship between the persons concerned is to be deciphered.

Example 1: Pointing to a photograph, a man said, "I have no brother or sister but that man's father is my father's son." Whose photograph was it?

(a) His own **(b)** His son's **(c)** His father's **(d)** His nephew's

Sol. Since the narrator has no brother, his father's son is he himself. So, the man who is talking is the father of the man in the photograph or the man in the photograph is his son.

Hence, the answer is **(b).**

Example 2: Ali introduces Ramiz as the son of the only brother of his father's wife. How is Ramiz related to Ali?

(a) Cousin **(b)** Son **(c)** Uncle **(d)** Son-in-law

Sol. The relations may be analyzed as follows: Father's wife — Mother; Mother's brother — Uncle; Uncle's son — Cousin. So, Ramiz is Ali's cousin. Hence, the answer is **(a)**

Example 3: A and B are brothers. C and D are sisters. A's son is D's brother. How is B related to C?

(a) Father **(b)** Brother **(c)** Grandfather **(d)** Uncle

Sol. Clearly, B is the brother of A; A's son is D's brother. This means D is the daughter of A. Since C and D are sisters, C is also the daughter of A. So, B is the uncle of C. Hence, the answer is **(d).**

Example 4: Given that

1. A is the mother of B; **2.** C is the son of A;

3. D is the brother of E; **4.** E is the daughter of B;

The grandmother of D is

(a) A **(b)** B **(c)** C **(d)** D

Sol. D is the brother of E and E is the daughter of B. This means that D is the son of B. Also, A is the mother of B. So, A is the grandmother of D. Hence, the answer is **(a).**

Example 5: A and B are married couple. X and Y are brothers. X is the brother of A. How is Y related to B ?

(a) Brother-in-law (b) Brother (c) Son-in-law (d) Cousin

Sol. A and B are husband and wife. Since X and Y are brothers, and X is the brother of A, Y is also the brother of A. Thus, Y is the brother-in-law of B. Hence, the answer is (a).

EXERCISE

1. **Pointing to a man on the stage, Jannat Hussain said, "He is the brother of the daughter of the wife of my husband." How is the man on the stage related to Jannat Hussain?**

   (a) Son (b) Husband (c) Cousin (d) Nephew

2. **Showing the man receiving the prize, Ahtesham Jan said, "He is the brother of my uncle's daughter." Who is the man to Ahtesham Jan?**

   (a) Son (b) Brother-in-law (c) Nephew (d) Cousin

3. **Pointing to a man, a woman said, "His mother is the only daughter of my mother." How is the woman related to the man?**

   (a) Mother (b) Daughter (c) Sister (d) Grandmother

4. **Pointing to a photograph, a person tells his friend, "She is the granddaughter of the elder brother of my father." How is the girl in the photograph related to this man?**

   (a) Niece (b) Sister (c) Aunt (d) Sister-in-law

5. **Pointing to a photograph, Syed Yasir Usman said, "She is the daughter of my grandfather's only son." How is Syed Yasir Usman related to the girl in the photograph?**

   (a) Father (b) Brother (c) Cousin (d) None of these

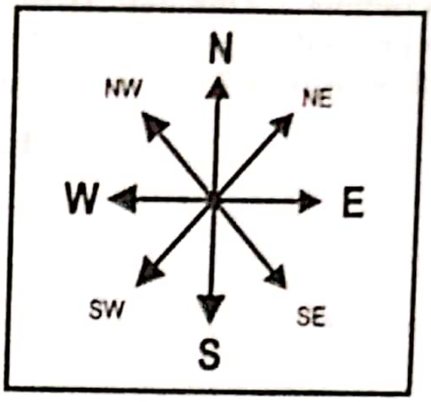
ANSWERS

1. **(a) :** Wife of husband — Herself; Brother of daughter — Son. So, the man is Jannat Hussain's son.
2. **(d) :** Brother of uncle's daughter — Uncle's son — Cousin.
3. **(a) :** Only daughter of my mother — Myself. So, the woman is man's mother.
4. **(a) :** Brother of father — Uncle; Uncle's granddaughter — daughter of uncle's son — daughter of cousin — niece.
5. **(b) :** My grandfather's only son-My father. So, the girl is the daughter of Syed Yasir Usman's father, i.e. Syed Yasir Usman is the girl's brother.

6- DIRECTION SENSE TEST

In this test, the questions consist of a sort of direction puzzle. A successive follow-up of directions is formulated and the candidate is required to ascertain the final direction or the distance between two points. There are four main directions (N, S, E and W) and four cardinal directions. They are:

- N-E (North-East)

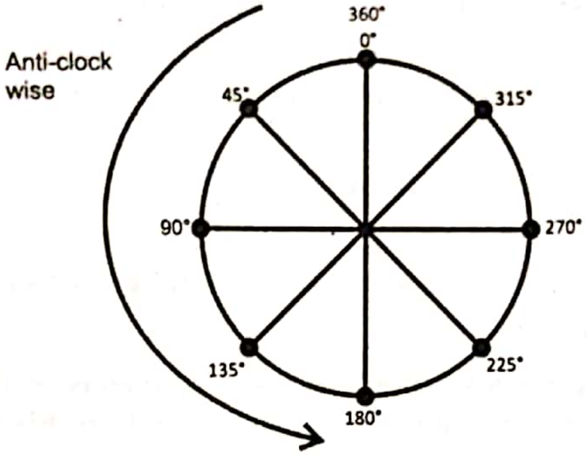


- N-W (North-West)
- S-E (South-East) and
- S-W (South-West)

We should use the diagram as given in question for the purpose of sensing directions.

Rotation of Angles

To solve angle movement questions it is necessary to know about the rotations of angles which are given below. Left turn will be Anti-clockwise direction and Right turn Clockwise direction

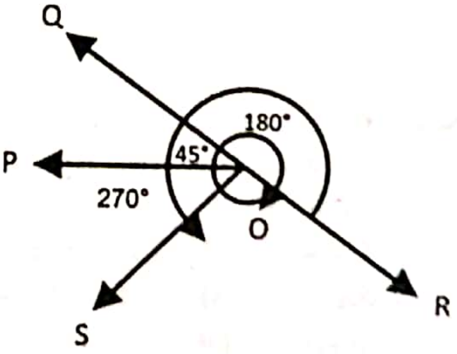


The Change in Direction when a Person or Vehicle Takes A Right or a Left Turn

Direction before taking the turn	Direction in which the person or vehicle will be moving after taking the turn	
	Right	**Left**
North	East	West
South	West	East
East	South	North
West	North	South

Example 1: A man is facing west. He turns 45° in the clockwise direction and then another 180° in the same direction and then 270° in the anticlockwise direction. Which direction is he facing now?

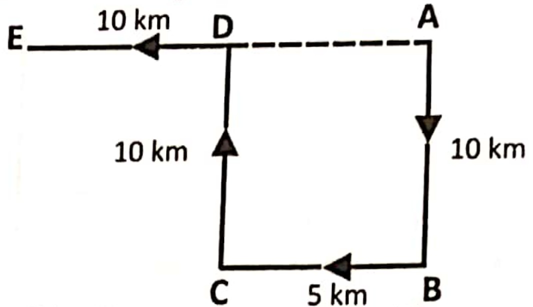
(a) South
(b) North-west
(c) West
(d) South-west



Sol. Clearly, the man initially faces in the direction OP. On moving 45° clockwise, he faces in the direction. OQ. On further moving 180° Clockwise, he faces in the direction OR. Finally, on moving 270° anticlockwise, he faces in the direction OS, which is Southwest.

Hence, the answer is **(d)**.

Example 2. One day, Mian Ghulam Qadir left home and cycled 10 km southwards, turned right and cycled 5 km and turned right and cycled 10 km and turned left and cycled 10 km. How many kilometers will he have to cycle to reach his home straight?



(a) 10 km (b) 15 km (c) 20 km (d) 25 km

Sol. Clearly, Mian Ghulam Qadir starts from home at A, moves 10 km southwards upto B, turns right and moves 5 km upto C, turns right again and moves 10 km upto D and finally turns left and moves 10 km upto E.

Thus, his distance from initial position A

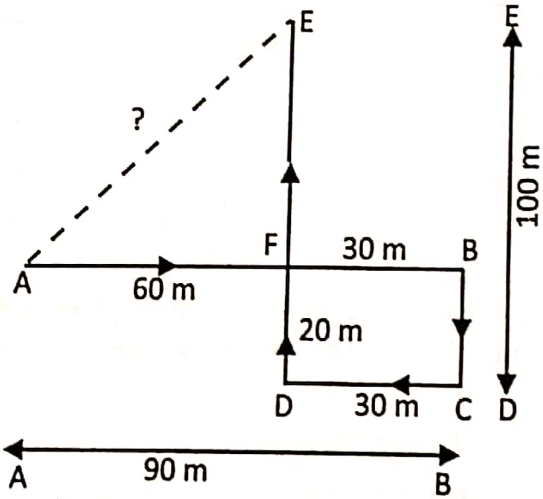
$$= AE = AD + DE$$

$$= BC + DE = (5 + 10) \text{ km} + 15 \text{ km}.$$

Hence, the answer is **(b)**.

Example 3: A child is looking for his father. He went 90 meters in the east before turning to right. He went 20 meters before turning to right again to look for his father at his uncle's place 30 meters from this point. His father was not there. From there, he went 100 meters to north before meeting his father in a street. How far did the son meet his father from starting point?

(a) 80 meters (b) 100 meters (c) 140 meters (d) 260 meters



Sol. Clearly, the child moves from A 90 m eastwards up to B, then turns right and moves 20 m up to C, then turns right and moves 30 m up to D. Finally, he turns right and moves 100 m up to E.

Clearly $BF = CD = 30$ m.

So, $AF = AB - BF = 60$ m.

Also, $DE = 100$ m,

$DF = BC = 20$ m.

So, EF = DE – DF = 80 m.

$\therefore$ His distance from starting point $A = AE = \sqrt{AF^2 + EF^2}$

$$= \sqrt{(60)^2 + (80)^2}$$

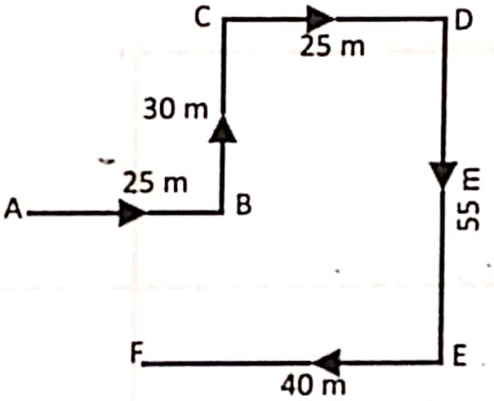
$$= \sqrt{3600 + 6400} = \sqrt{10000} = 100\,m.$$

Example 4. Karimdad Chugtai faces towards north. Turning to his right, he walks 25 meters. He then turns to his left and walks 30 meters. Next, he moves 25 meters to his right. He then turns to his right again and walks 55 meters. Finally, he turns to the right and moves 40 meters. In which direction is he now from his starting point?

(a) South-west (b) South (c) North-west (d) South-east

Sol. Karimdad turns towards right from north direction. So, he walks 25 m towards east up to B, turns left and moves 30 m up to C, turns right and goes 25 m up to D. At D, he turns to right towards the south and walks 55 m up to E. Next, he again turns to right and walks 40 m upto F, which is his final position. F is to the south-east of A. So, he is to the south-east from his starting point.

Hence, the answer is **(d)**.



EXERCISE

1. **Hassan is facing east. He turns 100° in the clockwise direction and then 145° in the anticlockwise direction. Which direction he is facing now?**

(a) East (b) North-east (c) North (d) South-west

2. **Aleem walks 20 m North. Then, he turns right and walks 30 m. Now, he turns right and walks 35 m. Now, turning left, he walks 15 m. Again, he turns lefts and moves 15 m. In which direction and how far is he from his original position?**

(a) 15 meters, East (b) 45 meters, East

(c) 15 meters, West (d) 45 meters, West

3. **A man starting from his home walks 5 km towards East, and then he turns left and goes 4 km. At last he turn to his left and walks 5 km. Now Find the distance between the man and his home and also find at which direction?**

(a) 4 kilometer North (b) 5 kilometer North-east

(c) 4 kilometer East (d) 14 kilometer West

4. **A man starting from his home moves 4 km towards East, then he turns right and moves 3 km. Now what will be the minimum distance covered by him to come back to his home?**

(a) 5 Km (b) 10 km (c) 20 km (d) 12 km

5. **Usman started from his home and moved 5 km to reach the crossing point of the palace. In which direction was Usman going, if the road opposite to his direction goes to the hospital. The road to the right of Usman goes to the station. If the road which goes to station is just opposite to the road of the IT-Park, then in which direction is Usman which leads to the IT- Park?**

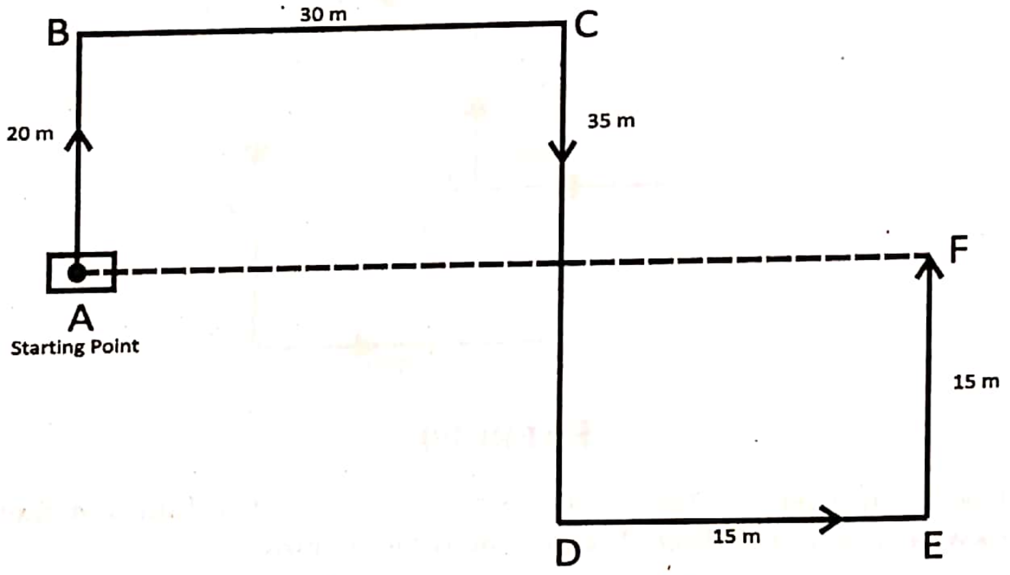
(a) Right (b) Left (c) North (d) South

ANSWERS

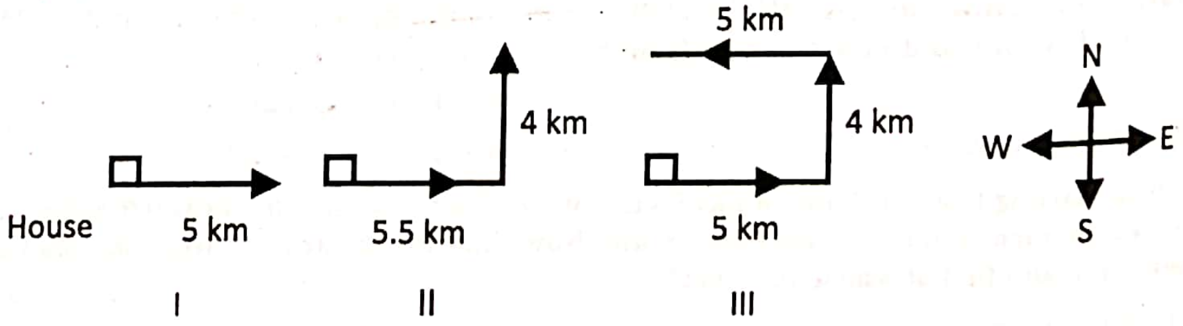
1. **(b):** Hassan man initially faces towards east, *i.e,* in the direction OA. On further moving 100° clockwise, he faces in the direction OB. On further moving 145° clockwise he faces in the direction OC. Clearly, OC makes an angle of (145° - 100°) *i.e.* 45° with OA and as such points in the direction North-east.

2. **(b):** Aleem's distance from his original position (AF) = BC + DE = 30 + 15 = 45 m

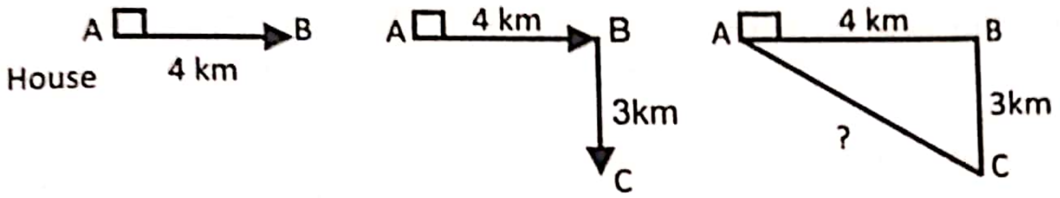
Also 'F' lies to East of 'A'. See the diagram below.



3. **(a):** From the diagram below we can find he is 4 km from his house and is in North Direction.



4. **(a):** Man starts from point A. As shown in diagram below.



We can calculator the distance by using formula $a^2+b^2=c^2$ (Pythagorean equation)

Minimum Distance = $AC=\sqrt{(AB)^2+(BC)^2}$

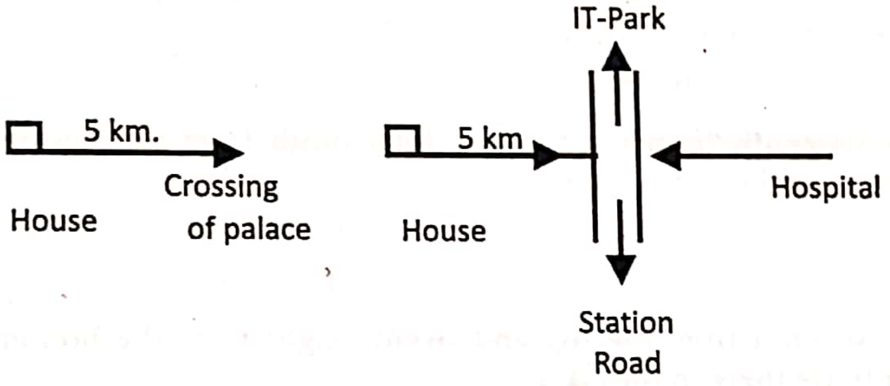
$$\text{Minimum distance} =\sqrt{(4)^2+(3)^2}$$

$$=\sqrt{16+9}$$

$$=\sqrt{25}$$

$$=5\,\text{km.}$$

5. **(b):** From the diagram below its shows that the road which goes towards the IT-Park is left of Usman.



II

7-Person Ranking Test

In this, generally the ranks of a person both from the top and from the bottom are mentioned and the total number of persons is asked. However, sometimes this question is put in the form of a puzzle of interchanging seats by two persons.

Example 1. Raza ranked ninth from the top and thirty eight from the bottom in a class. How many students are there in the class?

(a) 45 **(b)** 46 **(c)** 47 **(d)** 48

Sol. Clearly, the whole class consists of:

i. 8 students who have a rank higher than Raza;

ii. Raza; and

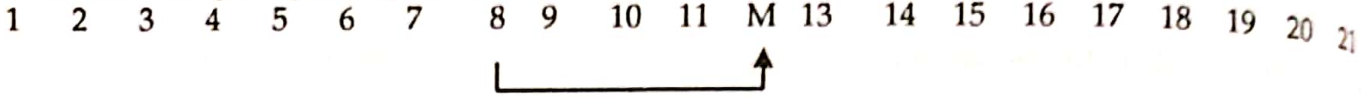
iii. 37 students who have rank lower than Raza.

i.e. (8+1+37) = 46 students. Hence, the answer is **(b)**.

Example 2. In a row of 21 girls, when Maryam was shifted by four place towards the right, she became 12th from the left end. What was her earlier position from the right end of the row?

(a) 9th (b) 10th (c) 11th (d) 14th

Sol. The change of place by Maryam can be shown as under:



Clearly, Maryam's earlier position was 8th from the left end and 14th from the right end. Hence, the answer is *(d)*.

EXERCISE

1. **1.In a row of trees, one tree is fifth from either end of the row. How many trees are there in the row?**

   (a) 8 (b) 9 (c) 10 (d) 11

2. **In a queue, Jannat Hussain is 10th from the front while Musab is 25th from behind and Khadija is just in the middle of the two. If there be 50 persons in the queue, what position does Khadija occupy from the front?**

   (a) 20th (b) 19th (c) 18th (d) 17th

3. **Jabar ranks sixteenth from the top and forty-ninth from the bottom in a class. How many students are there in the class?**

   (a) 64 (b) 65

   (c) 66 (d) Cannot be determined

4. **Saqib ranks seventh from the top and twenty eight from the bottom in a class. How many students are there in the class?**

   (a) 37 (b) 36 (c) 35 (d) 34

5. **If Atif finds that he is twelfth from the right in a line of boys and fourth from the left, how many boys should be added to the line such that there are 28 boys in the line?**

   (a) 12 (b) 13 (c) 14 (d) 20

ANSWERS

1. **(b) :** Clearly, number of trees in the row = (4+1+4) = 9.
2. **(c) :** Number of persons between Jannat Hussain and Musab = 50 - (10 + 25) = 15. Since Khadija lies in middle of these 15 persons, so Khadija's position is 8th from Jannat Hussain i.e. 18th from the front.
3. **(a)** : Clearly, number of students in the class = (15 + 1+ 48) = 64.
4. **(d)** : Clearly, number of students in the class = (6+1+27) = 34.
5. **(b) :** Clearly, number of boys in the line = (11+1+3) = 15.
   Number of boys to be added = 28 - 15 = 13

8-TIME SEQUENCE TEST

Example. 1. Sabir remembers that his brother's birthday is after fifteenth but before eighteenth of February whereas his sister Rabia remembers that her brother's birthday is after sixteenth but before nineteenth of February. On which day in February is Sabir's brother's birthday?

(a) 16 **(b)** 17 **(c)** 18 **(d)** 19

Sol. According to Sabir, the brother's birthday is on one of the days among 16th and 17th February. According to Rabia, the brother's birthday is on one of the days among 17th and 18th February. Clearly, Sabir's brother's birthday is on the day common to both the above groups, i.e. 17th February. Hence, the answer is **(b)**.

Example. 2. If it was Saturday on 17th December, 1982 what will be the day on 22nd December, 1984?

(a) Monday **(b)** Tuesday **(c)** Wednesday **(d)** Sunday

Clearly, every day repeats itself on the seventh day. Now, 17th Dec, 1982-17th Dec, 1983 is a period of 365 days. Dividing by 7, we get 52 weeks and one day. Thus, the 365th day will be the same as the first day i.e. 16th Dec. 1983 is also Saturday.

Now, 16th Dec, 1983-16th Dec, 1984 is a period of 365 days (because 1984, being a leap year, has 29 days in February). Thus, as shown above, 14th Dec. 1984 will be the same as 16th Dec. 1983, i.e. Saturday. So, 21st Dec. 1984 is also Saturday and thus, 22nd Dec. 1984 is a Sunday.

Hence, the answer is **(d)**.

Note : For such questions as Example 2. remember

(i) A year has 365 days.

(ii) Years, divisible by 4, are leap years, e.g. 1980, 1984, 1988, 1992, 1996, ...

They have 366 days.

(iii) February in a leap year has 29 days.

(iv) The last day of a year is the same as first day.

Thus, if the first day of a year is Friday, then the last day of the year is Friday and the first day of the next year is Saturday.

However, if the first day of a leap year is Friday, then the last day of the year is Saturday and the first day of the next year is Sunday.

EXERCISE

1. **Deedag Bizenjo remembers that his brother Aleem's birthday falls after 20th May but before 28th May, while Meesha remembers that Aleem's birthday falls before 22nd May but after 12th May. On what date Aleem's birthday falls?**

(a) 20th May **(b)** 21st May

(c) 22nd May **(d)** Cannot be determined

2. **Mahira Khan remembers that her father's birthday was certainly after eighth but before thirteenth of December. Her sister Natasha remembers that their father's birthday was definitely after ninth but before fourteenth of December. On which date of December was their father's birthday?**

(a) 10th
(b) 11th
(c) 12th
(d) Data inadequate

3. **Standing on a platform, Abid told Resham that Behria Town was more than ten kilometers but less than fifteen kilometers from there. Resham knew that it was more than twelve but less then fourteen kilometers from there. If both of them were correct, which of the following could be the distance of Behria Town from the platform?**

(a) 11 km
(b) 12 km
(c) 13 km
(d) 14 km

4. **Aftab Lagahri leaves his house at 20 minutes to seven in the morning, reaches Gul Khan,s house in 25 minutes, they finish their breakfast in another 15 minutes and leave for their office which takes another 35 minutes. At what time do they leave Gul Khan's house to reach their office?**

(a) 7.40 a.m
(b) 7.20 a.m
(c) 7.45 a.m
(d) 8.15 a.m

5. **Ajab Khan left home for the bus stop 15 minutes earlier than usual. It takes 10 minutes to reach the stop. He reached the stop at 8.40 a.m. What time does he usually leave home for the bust stop?**

(a) 8.30 a.m
(b) 8.45 p.m
(c) 8.55 a.m
(d) None of these

ANSWERS

1. **(b)** : According to Deedag Bizenjo, Aleem's birthday falls on one of the days among 21st, 22nd, 23rd, 24th, 25th, 26th and 27th May. According to Meesha, Aleem's birthday falls on one of the days among 13th, 14th, 15th, 16th, 17th, 18th, 19th, 20th, and 21st May. The day common to both the groups is 21st May.

   ∴ Aleem's birthday falls on 21st May.

2. **(d)** : According to Mahira Khan, the father's birthday falls on one of the days among 9th, 10th, 11th, and 12th-, December. According to Natasha, the father's birthday falls on one of the days among 10th, 11th, 12th and 13th December. The days common to both the groups are 10th, 11th and 12th December. So, the father's birthday falls on any one of these days.

3. **(c)**: Clearly, according to Resham, the distance was more than 12 kms but less than 14 kms, which is 13 kms.

4. **(b)** : Aqib leaves his house at 6.40 am. He reaches Gul Khan's house in 25 minutes, i.e. at 7.05 am. Both leave for office 15 minutes after 7.05 am, i.e. at 7.20 am.

5. **(*d*)** : Clearly, Ajab Khan left home 10 minutes before 8.40 am, i.e. at 8.30 am. But it was 15 minutes earlier than usual. So, he usually left for the stop at 8.45 am.

9-MATHEMATICAL OPERATION

In this type, you are provided with subsitutes for various mathematical symbols, followed by a question involving calculation of an expression or choosing the correct/incorrect equation. The candidate is required to put in the real signs in the given equation and then solve the questions as required.

Note: While solving a mathematical expression, proceed according to the rule BODMAS – i.e. Brackets, Of, Division, Multiplication, Addition, Subtraction.

Example. 1. If '+' mean 'divided by', '−' mean 'multiplied by', '×' means 'minus' and '÷' means 'plus', which of the following will be the value of the expression $16 \div 8 - 4 + 2 \times 4$?

(a) 16 (b) 28 (c) 32 (d) 44

Sol. Putting the proper signs in the given expression , we get:

$16+8\times4\div2-4 = 16+8\times2-4 = 16+16-4 = 32-4 = 28$

So, the answer is (b).

Example. 2. If + means ÷, − means ×, ÷ means + and × means −, then

$36 \times 12 + 4 \div 6 + 2 - 3 = ?$

(a) 2 (b) 18 (c) 42 (d) $6\frac{1}{2}$

Sol. Using the proper signs, we get:

$36 - 12 \div 4 + 6 \div 2 \times 3 = 36 - 3 + 3 \times 3 = 36 - 3 + 9 = 36+6 = 42$

So, the answer is (c).

Example. 3. If A means 'plus', B means 'minus', C means 'divided by' and D means 'multiplied by', then 18 A 12 C 6 D 2 B 5 = ?

(a) 15 (b) 25 (c) 17 (d) 45

Sol. Using the proper signs, we get:

Given expression

$= 18 + 12 \div 6 \times 2 - 5 = 18 + 2 \times 2 - 5$

$= 18 + 4 - 5 = 22 - 5 = 17$

So, the answer is (c).

Example. 4. If × stands for −, ÷ stands for +, + stands for ÷ and − stands for ×, which one of the following equations is correct ?

(a) $15-5\div5\times20+10=6$ (b) $8\div10-3+5\times6=8$

(c) $6\times2+3+12-3=15$ (d) $3\div7-5\times10+3=10$

Sol. Using the proper signs, we get:

Expression in (a) $= 15 \times 5 + 5 - 20 \div 10 = 15 \times 5 + 5 - 2 = 75 + 5 - 2 = 78.$

Expression in (b) $= 8 + 10 \times 3 \div 5 - 6 = 8 + 10 \times \frac{3}{5} - 6 = 8 + 6 - 6 = 8$

Expression in (c) $= 6 \times 2 + 3 + 12 \times 3 = 6 - \frac{2}{3} + 36 = 42 - \frac{2}{3} = \frac{124}{3}$

Expression in **(d)** $= 3 \div 7 \times 5 - 10 \div 3 = 3 + 7 \times 5 - \frac{10}{3} = 3 \div 35 - \frac{10}{3} = \frac{104}{3}$

∴ Statement **(b)** is true.

Example. 5. It being given that : > denotes +, < denotes–, + denotes÷, – denotes =, = denotes 'less than' and x denotes 'greater than', find which of the following is a correct statement.

(a) $3 \div 2 \div 4 < 9 \div 3 - 2$ **(b)** $3 \div 2 \div 4 < 18 \div 3 - 1$

(c) $3 + 2 - 4 > 8 \div 4 - 2$ **(d)** $3 \div 2 - 4 > 9 \div 3 - 3$

Sol. Using proper notations, we have:

(a) Given statement is $3 \div 2 \div 4 < 9 \div 3 - 2$ or $\frac{11}{2} < 1$, which is not true.

(b) Given statement is $3 \div 2 \div 4 < 18 \div 3 - 1$ or $9 < 5$, which is not true.

(c) Given statement is $3 \times 2 - 4 > 8 \div 4 - 2$ or $1 > 0$, which is true.

(d) Given statement is $3 \div 2 - 4 > 9 \div 3 - 3$ or $-\frac{5}{2} > 0$, which is not true.

So, the statement **(c)** is true.

10-ARITHMETICAL REASONING

Example. 1. In a chess tournament each of six players will play every other player exactly once. How many matches will be played during the tournament?

(a) 12 **(b)** 15 **(c)** 30 **(d)** 36

Sol. Clearly, we will consider the following matches:

i. matches of first player with other 5 players;

ii. matches of second player with 4 players other than the first player;

iii. matches of third player with 3 players other than the first and second player;

iv. matches of fourth player with 2 players other than the first three players; and

v. match of fifth player with the sixth one.

So, number of matches played during the tournament = 5+4+3+2+1= 15. Here answer is (b)

Example. 2. A man has a certain number of small boxes to pack into parcels. If he packs 3, 4, 5 or 6 in a parcel, he is left with one over; if he packs 7 in a parcel, none is left over. What is the number of boxes, he may have to pack?

(a) 106 **(b)** 301 **(c)** 309 **(d)** 400

Sol. Clearly, the required number would be such that it leaves a remainder of 1 when divided by 3, 4, 5 or 6 and no remainder when divided by 7. Such a number is 301.

Hence, the answer is **(b).**

Example. 3. A, B, C and D play a game of cards. A says to B, "If I give you 8 cards, you will have as many as C has and I shall have 3 less than what C has. Also, if I take 6 cards from C, I shall have twice as many as D has. " If B and D together have 50 cards, how many cards has A got?

(a) 40 **(b)** 37 **(c)** 27 **(d)** 23

Sol. Clearly, we have

$B + 8 = C$ (i) $A - 8 = C - 3$ (ii)

$A + 6 = 2D$ (iii) $B + D = 50$ (iii)

Putting $C = A - 5$ from (*ii*) into (*i*), we have:

$B + 8 = A - 5$ or $A + B = 13$ (iv)

Putting $D = 50 - B$ from (vi) into (iii), we have: (v)

$A + 6 = 100 - 2B$ or $A + 2B = 94$ (vi)

Solving (*v*) and (*vi*), we get $B = 27$ and $A = 40$.

$\therefore$ A has 40 cards.

Hence, the answer is **(a)**.

Example. 4. In a group of cows and hens, the number of legs are 14 more than twice the number of heads. The number of cows is?

(a) 5 **(b)** 7 **(c)** 10 **(d)** 12

Sol. Let the number of cows be x and the number of hens be y. Then, number of legs in the group = $4x + 2y$

number of heads in the group= $x + y$.

So, $4x + 2y = 2(x + y) + 14$ or $4x + 2y = 2x + 2y + 14$ or $2x = 14$ or $x = 7$.

$\therefore$ Number of cows = 7.

Hence, the answer is **(b)**.

Example. 5. A worker may claim Rs. 15 for each km which he travels by taxi and Rs. 5 for each km which he drives his own car. If in one week he claimed Rs. 500 for travelling 80 km how many kms did he travel by taxi?

(a) 10 **(b)** 20 **(c)** 30 **(d)** 40

Sol. Let the distance covered by taxi be x km.

Then, distance covered by car = $(80 - x)$ km.

$15x + 5(80 - x) = 500$ or $15x + 400 - 5x = 500$ or $10x = 100$ or $x = 10$.

$\therefore$ Distance covered by taxi = 10km.

Hence, the answer is **(a)**.

CHAPTER 18

LOGICAL REASONING

The word '**Logic**' is derived from the Greek noun 'logos' meaning both 'thought' and 'the word expressing thought'. Thus, LOGIC is the 'science of thought as expressed in language'. This means that the questions on logic are to be solved as per the information given without any concern of the formal validity or truth of the statements, i.e. conclusion should follow directly from the statements given. With this unique characteristic, the logic test becomes an instrument of teaching the candidates to follow the rules and work as per the instructions without an error. Thus, it prepares the mind for all types of reasoning practices and teaches how to detect and avoid mistakes in the same.

Reasoning is the capacity for a person to make sense of things to establish & verify facts, to rationally work through data, information, facts, and beliefs. It is the process of forming conclusions and judgments from facts or premises. To put it plain and simple; it is the ability to coherently think from perceived premise to a logical conclusion.

There are two main types of reasoning: Inductive Reasoning and Deductive Reasoning.

Deductive Reasoning– Deductive reasoning is the form of reasoning in which a conclusion follows logically and coherently from the factual premises and proposition. These deductive arguments are based upon the concept of sound and consistent reasoning. If the premises are true, than the systematic reasoning with a constructed syllogism is considered valid in a deductive argument in making its conclusion certain with a degree of logical certainty. Plainly speaking. Deductive reasoning is the rationality of reasoning from pure logic. It is considered sound and pure logic.

Inductive Reasoning– Inductive reasoning is a form of reasoning that uses analogies, examples, observations, and experiences to form conclusive propositions. Inductive logic also uses experiences to formulate statements based on general observations of recurring patterns in nature, science, and everyday occurrences pulling from such things as samples cases, experiments, and natural eye observations. It is used mostly to explain properties and relations to objects or types based on previous observations. It must be understood that inductive arguments do not try to establish their conclusions through absolute certainty, but through observable and predictive certainty.

What is a Statement?

Statements are things that are said that are either true or false. They are claims. Here is one: "Neptune has the fastest winds in the solar system." A statement that is especially important might be called a ***proposition, assertion, judgment, hypothesis, principle, thesis***, or, in some situations, a *law*.

Statements have to be capable of being true or false. So, if you say, "Is it midnight?" then you've not made a statement. *Suggestions, commands, and proposals aren't statements either.* The suggestion "I suggest we should get a new refrigerator," and the command, "Stand back!" and the proposal, "Let's quit studying," are not statements. It would be very odd to call any of them "true" or "false." The following *are* statements: "She suggested we should get a new refrigerator," and "He commanded us to stand back," and "He proposed that we quit studying." These statements are all reports about someone's suggestion or command or proposal.

What is an Argument?

The word argument has more than one meaning. It takes only one person to have an argument, not two. Saying that two people are "in an argument" means that there are two arguments, not one, in our sense of "argument." Each of the two has his or her own argument. In short, our word argument is a technical term with a more precise meaning than it has in ordinary conversation. Statements that serve as reasons in an argument are also called **premises**. Any argument must have one or more premises. And it will have one or more "inference steps" taking you from the premises to the conclusion. The simplest arguments have just one step. Here is an example of a very simple argument that takes you to the conclusion in just one inference step from two premises:

If is raining, we should take the umbrella.

It is raining.

So, we should take the umbrella.

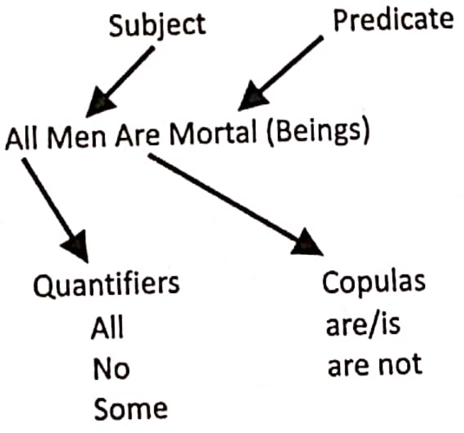
LOGICAL DEDUCTION

The phenomenon of deriving a conclusion from a single proposition or a set of given propositions, is known as **logical deduction.**

The Proposition consists of three main parts:

1. **Subject:** The Subject is that about which something is said.
2. **Predicate:** The Predicate is the part of the Proposition denoting that which is affirmed or denied about the subject.
3. **Copula:** The Copula is that part of the Proposition which denotes the relation between the Subject and the Predicate.

Consider the Proposition '*Man is cultured*'.



Here an information is given about the man. So 'Man' is the Subject. 'Cultured' is th quality affirmed for this Subject. So it is the Predicate. 'is' denotes the relation between the Subject and the President. So, it is the Copula. Here is another example, All men are mortal

Quantifiers can also be used with subject and these can be all, no, some, almost etc.

Classification of Propositions

'Propositions' can be classified into four types:

- **Universal affirmative** proposition or **Type- A. Example:** All Snakes are reptiles.
- **Universal Negative** Proposition or **Type-E. Example:** No boy is Intelligent
- **Particular Affirmative** or **Type-I. Example:** some men are foolish.
- **Particular Negative** or **Type-O. Example:** some animals are not wild

Summarizing the above Propositions

Statement form	**Quantity**	**Quality**	**Distribution**
All S is P	Universal	Affirmative	S only
No S is P	Universal	Negative	Both S and P
Some S is P	Particular	Affirmative	Neither S nor P
Some S is not P	Particular	Negative	Only P
S: Subject; P= Predicate			

Two Inferential Processes of Deduction:

I. Immediate Deductive Inference:

Here, conclusion is deduced from one of the given propositions, by any of the three ways - **conversion, obversion** and **contraposition.**

1. **Conversion:** The Conversion proceeds with interchanging the subject term and the predicate term i.e. the subject term of the premise becomes the predicate term of the conclusion and the predicate term of the premise becomes the subject of the conclusion. The given proposition is called convertend, whereas the conclusion drawn from it is called converse.

Table of Valid Conversions

Statement form	**Valid Conversion/ Conclusion**	**Examples**
All S is P	Some P is S	Statement: All men are fools Valid Conclusion: some fools are men
No S is P	No P is S	Statement: No men are fools Valid Conclusion: No fools are men
Some S is P	Some P is S	Statement: Some men are fools Valid Conclusion: Some fools are men
Some S is not P	No valid Conversion	Statement: Some men are not fools Valid Conclusion:---
S: Subject; P= Predicate		

Note that in a conversion, the quality remains the same and the quantity may change.

2. **Obversion:** In obversion, we change the quality of the proposition and replace the predicate term by its complement.

Table of Valid Obversions

Obvertend	Obverse
A: All birds are mammals.	E: No birds are non-mammals.
E: No poets are singers.	A: All poets are non-singers.
I: Some nurses are doctors.	O: Some nurses are not non-doctors.
O: some politicians are not statesmen.	I: Some politicians are non-statesmen.

3. **Contraposition:** To obtain the contrapositive of a statement, we first replace the subject and predicate terms in the proposition and then exchange both these terms with their complements.

Table of Valid Contrapositions

Proposition	Contrapositive
A: All birds are mammals.	A: All non-mammals are non-birds.
I: Some birds are mammals.	I: Some non-mammals are non-birds.

Note: The valid converse, obverse or contrapositive of a given proposition always logically follows from the proposition.

II. Mediate Deductive Inference (SYLLOGISM):

First introduced by Aristotle, a Syllogism is a deductive argument in which conclusion has to be drawn from two propositions referred to as the premises.

Example

1. All lotus are flowers.
2. All flowers are beautiful.
3. All lotus are beautiful.

Clearly, the propositions 1 and 2 are the Premises and the proposition 3, which follows from the first two propositions, is called the Conclusion.

Term

In Logic, a TERM is a word or a combination of words, which by itself can be used as a subject or predicate of a proposition. Syllogism is concerned with three terms:

1. **Major Term**

It is the *predicate of the conclusion* and is denoted by **P** (first letter of 'Predicate').

2. **Minor Term**

It is the *subject of the conclusion* and is denoted by **S** (first letter of 'Subject').

3. **Middle Term**

It is the *term common to both the premises* and is denoted by **M** (first letter of 'Middle'). Note that the middle term does not occur in the conclusion.

Example: Premises

1. All dogs are animals.
2. Tiger is a dog.

Conclusion: Tiger is an animal.

Here, 'animal' is the predicate of the conclusion and so, it is the Major Term, P.

'Tiger' is the subject of the conclusion and so, it is the Minor Term, S.

'Dog' is the term common to both the premises and so, it is the Middle Term, M.

Major and Minor Premise: Of the two premises, the *major premise* is that in which the middle term is the subject and the *minor premise* is that in which the middle term is the predicate.

Rules for Deriving the Conclusion

1. The conclusion does not contain the middle term.

Example

Statements: 1. All men are girls.
2. Some girls are students.

Conclusion: 1. All girls are men.
2. Some students are girls.

Since both the conclusions 1 and 2 contain the middle term 'girls', so neither of them can follow.

2. No term can be distributed in the conclusion unless it is distributed in the premises.

Example:

Statements: 1. Some dogs are goats.
2. All goats are cows.

Conclusions: 1. All cows are goats.
2. Some dogs are cows.

Statement 1 is an I type proposition which distributes neither the subject nor the predicate. Statement 2 is an A type proposition which distributes the subject, i.e. 'goats' only.

Conclusion 1 is an A type proposition which distributes the subject 'cow' only.

Since the term 'cow' is distributed in conclusion 1 without being distributed in the premises, so conclusion 1 cannot follow.

3. The middle term (M) should be distributed at least once in the premises. Otherwise, the conclusion cannot follow.

For the middle term to be distributed in a premise,

(i) M must be the Subject if premise is an A proposition.

(ii) M must be Subject or Predicate if premise is an E proposition.

(iii) M must be Predicate if premise is an O proposition.

Note that in an I proposition, which distributes neither the Subject nor the Predicate, the middle term cannot be distributed.

Example

Statements: **1.** All fans are watches.

2. Some watches are black.

Conclusions: **1.** All watches are fans.

2. Some fans are black.

In the premises, the middle term is 'watches'. Clearly, it is not distributed in the first premise which is an A proposition as it does not form its subject. Also, it is not distributed in the second premise which is an I proposition. Since the middle term is not distributed at least once in the premises, so no conclusion follows.

4. No conclusion follows

(a) *if both the premises are particular*

Example

Statements: **1.** Some books are pens.

2. Some pens are erasers.

Conclusions: **1.** All books are erasers.

2. Some erasers are books.

Since both the premises are particular, no conclusion follows.

(b) *if both the premises are negative*

Example

Statements: **1.** No flower is mango.

2. No mango is cherry.

Conclusions: **1.** No flower is cherry.

2. Some cherries are mangoes.

Since both the premises are negative, neither conclusion follows.

(c) *if the major premise is particular and the minor premise is negative.*

Example

Statements: **1.** Some dogs are bulls.

2. No tigers are dogs.

Conclusions: **1.** No dogs are tigers.

2. Some bulls are tigers.

Here the first premise containing the middle term 'dogs' as the Subject is the major premise and the second premise containing the middle term 'dogs' as the Predicate is the minor premise. Since the major premise is particular and the minor premise is negative, so no conclusion follows.

5. If the middle term is distributed twice, the conclusion cannot be universal.

Example

Statements: **1.** All fans are chairs.

2. No tables are fans.

Conclusions: 1. No tables are chairs.
2. Some tables are chairs.

Here, the first premise is an A proposition and so, the middle term 'fans' forming the subject is distributed. The second premise is an E proposition and so, the middle term 'fans' forming the predicate is distributed. Since the middle term is distributed twice, so the conclusion cannot be universal.

6. If one premise is negative, the conclusion must be negative.

Example

Statements: 1. All grasses are trees.
2. No tree is shrub.

Conclusions: 1. No grasses are shrubs.
2. Some shrubs are grasses.

Since one premise is negative, the conclusion must be negative. So, conclusion 2 cannot follow.

7. If one premise is particular, the conclusion is particular.

Example

Statements: 1. Some boys are thieves.
2. All thieves are dacoits.

Conclusions: 1. Some boys are dacoits.
2. All dacoits are thieves.

Since one premise is particular, the conclusion must be particular. So, conclusion 2 cannot follow.

8. If both the premises are affirmative, the conclusion would be affirmative.

Example

Statements: 1. All women are mothers.
2. All mothers are sisters.

Conclusions: 1. All women are sisters.
2. Some women are not sisters.

9. If major premise be affirmative, the conclusion must be particular.

Example

Statements: 1. All plays are stories.
2. Some poems are plays.

Conclusions: 1. Some poems are stories.
2. All stories are poems.

The first premise containing the middle term 'plays' as the subject is the major premise. Also, it is affirmative. So, the conclusion must be particular. Hence, conclusion 2 cannot follow.

VERIFICATION THROUGH VENN DIAGRAMS

Let us study the various possible cases and draw all possible inferences in each case, along with verification through Venn diagrams.

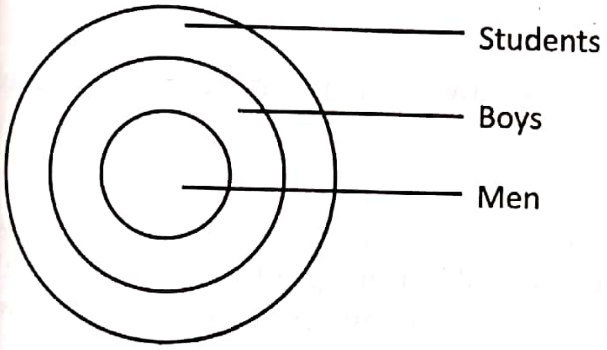
Case 1: All men are boys. All boys are students.

Immediate Deductive Inferences

The converse of first premise i.e. 'Some boys are men' and the converse of second premise i.e. 'Some students are boys' both hold.

Mediate Deductive Inferences

Since both the premises are universal and affirmative, the conclusion must be universal affirmative. Also, the conclusion should not contain the middle term. So, it follows that 'All men are students'. The converse of this conclusion i.e. 'Some students are men' also holds.

Venn diagram	Inferences
	1. Some boys are men. 2. Some students are boys. 3. All men are students 4. Some students are men.

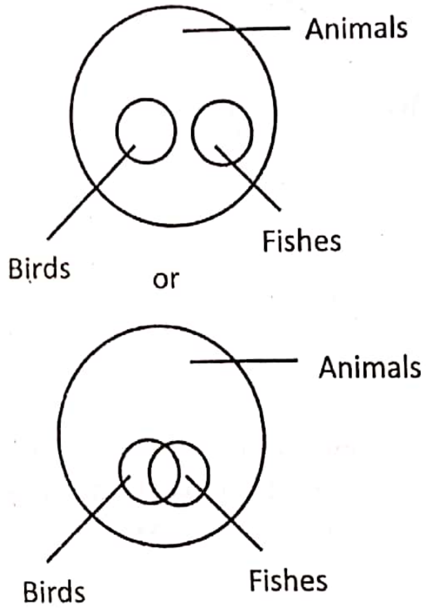
Case 2: All birds are animals. All fishes are animals.

Immediate Deductive Inferences

The converse of first premise i.e. 'Some animals are birds' and the converse of second premise i.e. 'Some animals are fishes' both hold.

Mediate Deductive Inferences

Both, being A-type propositions, distribute subject only. Thus, the middle term 'animals' is not distributed even once in the premises. So, no definite conclusion follows.

Venn diagram	Inferences
 Animals Birds Fishes or Animals Birds Fishes	1. Some animals are birds. 2. Some animals are fishes. 3. Either 'No bird is fish' or 'Some birds are fishes' as E and I-type propositions form a complementary pair

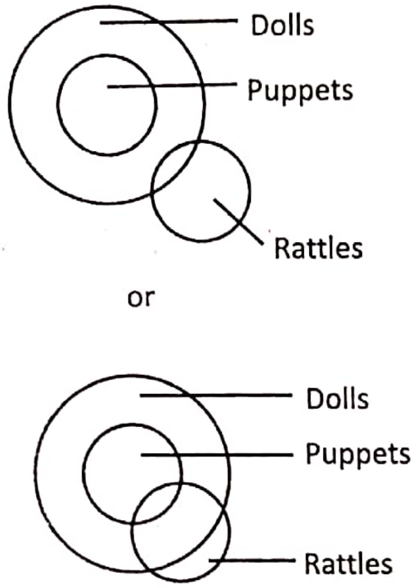
Case 3: All puppets are dolls. Some dolls are rattles.

Immediate Deductive Inferences

The converse of the first premise i.e. 'Some dolls are puppets' and the converse of the second premise i.e. 'Some rattles are dolls', both hold.

Mediate Deductive Inferences

First premise, being an A-type proposition, distributes the subject only while the second premise, being an I-type proposition, distributes neither subject nor predicate. Since the middle term 'dolls' is not distributed even once in the premises, so no definite conclusion can be drawn.

Venn diagram	Inferences
 Dolls Puppets Rattles or Dolls Puppets Rattles	1. Some dolls are puppets. 2. Some rattles are dolls. 3. Either 'No puppet is rattle ' or ' Some puppets are rattles' as E and I-type propositions form a complementary type.

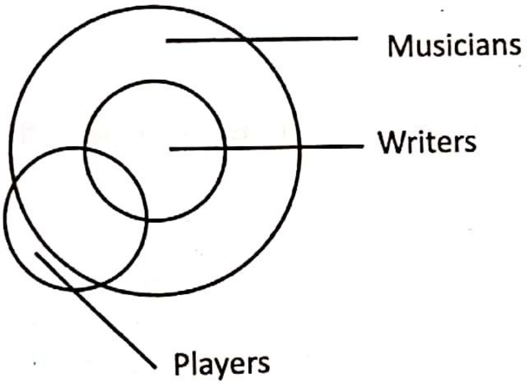
Case 4: Some writers are players. All players are musicians.

Immediate Deductive Inferences

The converse of the first premise i.e. 'Some players are writers' and the converse of the second premise i.e. 'Some musicians are players', both hold.

Mediate Deductive Inferences

Since one premise is particular, the conclusion must be particular and should not contain the middle term. So, it follows that 'Some writers are musicians'. The converse of this conclusion i.e. 'Some musicians are writers' also holds.

Venn diagram	Inferences
	1. Some players are writers. 2. Some musicians are players. 3. Some writers are musicians. 4. Some musicians are writers.

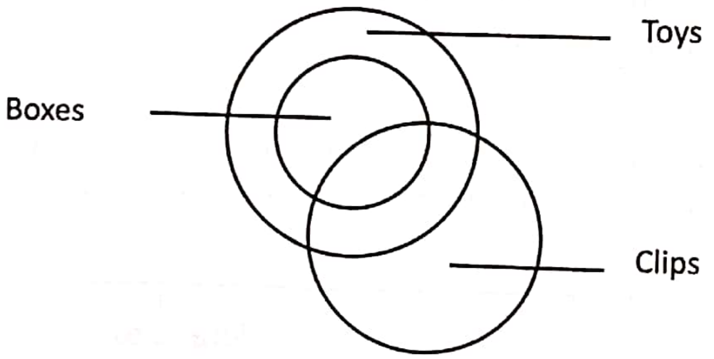
Case 5: All boxes are toys. Some boxes are clips.

Immediate Deductive Inferences

The converse of the first premise i.e. 'Some toys are boxes' and the converse of the second premise i.e. 'Some clips are boxes', both hold.

Mediate Deductive Inferences

Since one premise is particular, the conclusion must be particular and should not contain the middle term. So, it follows that 'Some toys are clips'. The converse of this conclusion i.e. 'Some clips are toys' also holds.

Venn diagram	Inferences
	1. Some toys are boxes. 2. Some clips are boxes. 3. Some toys are clips. 4. Some clips are toys.

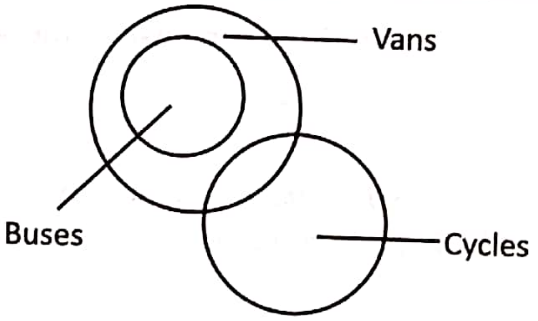
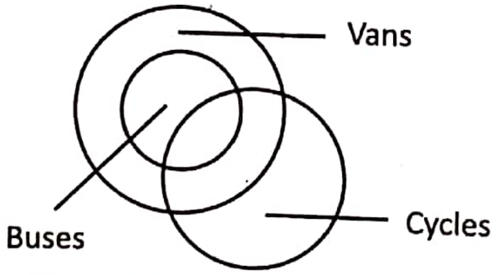
Case 6: All buses are vans. Some cycles are vans.

Immediate Deductive Inferences

The converse of the first premise i.e. 'Some vans are buses' and the converse of the second premise i.e. 'Some vans are cycles', both hold.

Mediate Deductive Inferences

First premise, being an A-type proposition, distributes subject only and the second premise, being an I-type proposition, distributes neither subject nor predicate. So, the middle term 'vans' is not distributed even once in the premises. Hence, no definite conclusion can be drawn.

Venn diagram	Inferences
 or 	1. Some vans are buses. 2. Some vans are cycles. 3. Either 'No bus is cycle' or 'Some buses are cycles' follows.

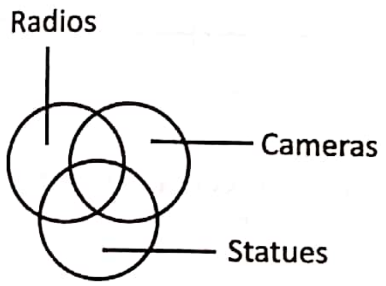
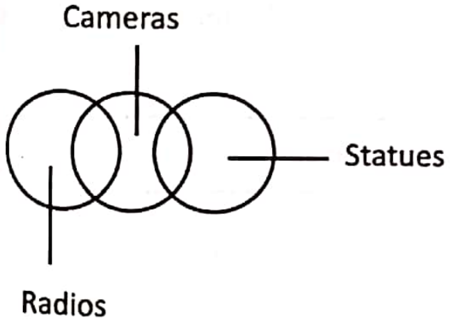
Case 7: Some radios are cameras. Some cameras are statues.

Immediate Deductive Inferences

The converse of the first premise i.e. 'Some cameras are radios' and the converse of the second premise i.e. 'Some statues are cameras', both hold.

Mediate Deductive Inferences

Since both premises are particular, no definite conclusion follows.

Venn diagram	**Inferences**
 or 	1. Some cameras are radios. 2. Some statues are cameras. 3. Either ' Some radios are statues' or 'No radio is statue' follows; as I and E-type Propositions form a complementary pair.

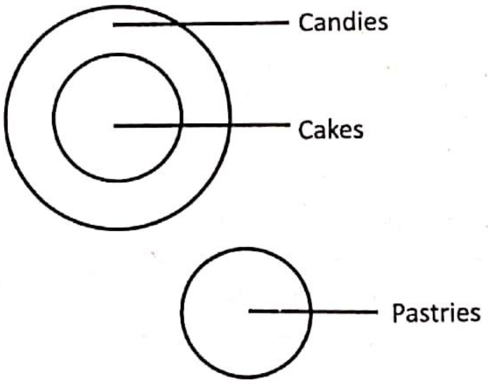
Case 8: All cakes are candies. No candy is pastry.

Immediate Deductive Inferences

The converse of the first premise i.e. 'Some candies are cakes' and the converse of the second premise i.e. 'No pastry is candy', both hold.

Mediate Deductive Inferences

Since both premises are universal, the conclusion must be universal. Since one premise is negative, the conclusion must be negative. So, it follows that 'No cake is pastry'. The converse of this conclusion i.e. 'No pastry is cake' also holds.

Venn diagram	Inferences
	1. Some candies are cakes. 2. No pastry is candy. 3. No cakes is pastry. 4. No pastry is cake.

Case 9: No coin is ring. All rings are bangles.

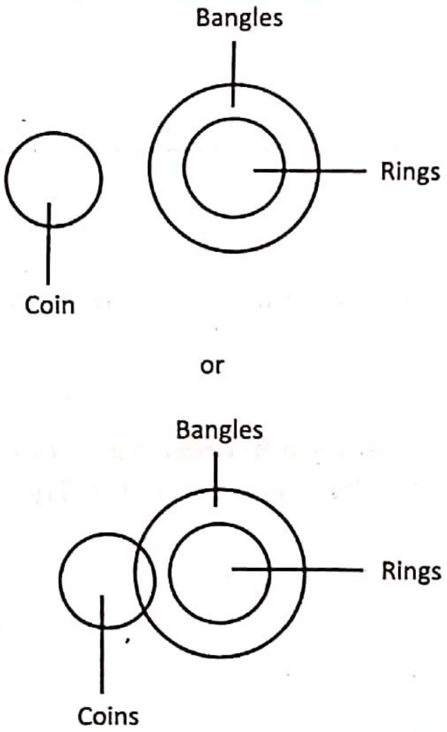
Immediate Deductive Inferences

The converse of the first premise i.e. 'No ring is coin' and the converse of the second premise i.e.'Some bangles are rings', both hold.

Mediate Deductive Inferences

First premise, being an E-type proposition, distributes both the subject and the predicate.

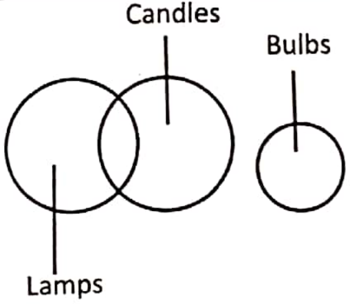
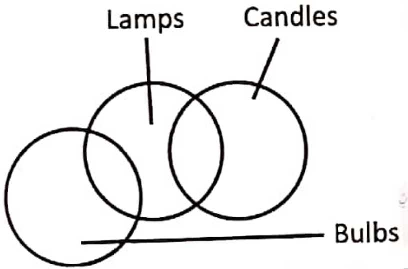
Second premise, being an A-type proposition, distributes the subject. Thus, the middle term 'ring' is distributed twice in the premises. So, the conclusion cannot be universal. Also, since one premise is negative, the conclusion must be negative. Thus, the conclusion must be particular negative i.e. O-type. So, it follows that 'some bangles are not coins'.

Venn diagram	Inferences
	1. No ring is coin. 2. Some bangles are rings. 3. Some bangles are not coins. 4. Either 'Some coins are bangles' or 'No coin is bangle' follows as E and I-type proposition forma complementary pair.

Case 10: Some lamps are candles. No candle is bulb.

Immediate Deductive Inferences

The converse of the first premise i.e. 'Some candles are lamps' and the converse of the second premise i.e. 'No bulb is candle', both hold.

Venn diagram	Inferences
 or 	1. Some candles are lamps. 2. No bulb is candle. 3. Some lamps are not bulbs. 4. Either 'Some lamps are bulbs' or 'No lamp is a bulb' follows E and I-type propositions form a complementary pair.

Mediate Deductive Inferences

Since one premise is particular and the other negative, the conclusion must be particular negative i.e. O-type, So, it follows that 'Some lamps are not bulbs'.

Important Points to Remember

While deriving logical conclusions, always remember that the following conclusions hold :

1. The converse of each of the given premises;
2. The conclusion that directly follows from the given premises in accordance with the rules of syllogism;
3. The converse of the derived conclusions.

EXERCISE

Directions: *In each question below are given two statements followed by two conclusions numbered I and II. You have to take the given two statements to be true even if they seem to be at variance from commonly known facts. Read the conclusions and then decide which of the given conclusions logically follows from the two given statements, disregarding commonly known facts.*

Give answer (a) if only conclusion I follows; (b) if only conclusion II follows; (c) if either I or II follows; (d) if neither I nor II follows and (e) if both I and II follow.

1. **Statements:** All planets are moons.
All moons are stars.

Conclusions: I. All moons are planets.
II. All planets are stars.

2. **Statements:** All men are dogs.
All dogs are cats.
Conclusions: I. All men are cats.
II. All cats are men.

3. **Statements:** All tubes are handles.
All cups are handles.
Conclusions: I. All cups are tubes.
II. Some handles are not cups.

4. **Statements:** All bags are cakes.
All lamps are cakes.
Conclusions: I. Some lamps are bags.
II. No lamp is bag.

5. **Statements:** All flowers are stems.
All stems are roots.
Conclusions: I. All roots are flowers.
II. All stems are flowers.

ANSWERS

1. **(b)** : Since both the statements are affirmative, the conclusion must be affirmative. However, conclusion I cannot follow as it contains the middle term. So, only conclusion II follows.
2. **(a)** : Since both the premises are affirmative, the conclusion must be affirmative. However, conclusion II being an A type proposition, distributes the term 'goats'. Since the term 'goats' is distributed in II without being distributed in any of the premises, so conclusion II cannot follow. Thus, only I follows.
3. **(d)**: Both the premises are A type propositions. So, in either, the middle term 'handles' forming the predicate is not distributed.

   Since the middle term is not distributed even once in the premises, so no conclusion follows.
4. **(d)**: Both the premises being A type propositions, the middle term 'cakes' forming the predicate is not distributed in any of them.

   Since the middle term is not distributed even once in the premises, so no conclusion follows.
5. **(d)**: Conclusion I being an A type proposition, distributes the term 'roots'. Since the term 'roots' is distributed in I without being distributed in any of the premises, so conclusion I cannot follow. Conclusion II cannot follow as it contains the middle term.

WHEN MORE THAN TWO CONCLUSIONS ARE GIVEN

In this type of questions, two statements called premises are given, followed by four conclusions. The candidate is required to find out which of the conclusions logically follow from the given premises. More than one conclusion may also follow. In such questions, first the given statements are analysed. If the middle term is not distributed even once, no conclusion follows. Thus, the middle term must be distributed at least once. To derive the correct conclusions, we usually take the help of Venn diagrams. Also, no conclusion follows in the following cases:

(i) If both the premises are particular.

(ii) If both the premises are negative.

However, in some cases, more than one Venn diagrams may be possible. In such cases, all the possible Venn diagrams are drawn and the solution is derived from each of these separately. Finally, the solution common to all the diagrams is taken as the answer.

Example 1:

Statements: Some flowers are apples.
Some apples are stones.

Conclusions:
I. No flower is stone.
II. All apples are stones.
III. Some stones are flowers.
IV. No apple is flower.

(a) Only either I or III follow
(b) Only I and IV follow
(c) Only II and III follow
(d) Only I, III and IV follow
(e) None follows

Sol. Since both the statements and premises are particular, no conclusion follows.

Hence, the answer is **(e)**.

Example 2: Statements: All leaves are papers.
Some papers are pictures.

Conclusions:
I. All pictures are papers.
II. All pictures are leaves.
III. Some leaves are pictures.
IV. No leaf is a picture.

(a) Only I, III and IV follows
(b) Only II and III follow
(c) Only III and IV follow
(d) Only III follows
(e) None of these

Sol. Clearly, here the middle term is 'papers'. Now, the first premise is an A type proposition and distributes the subject only. So, the middle term 'papers' forming the predicate, is not distributed. The second premise is an I type proposition and distributes neither the subject nor the predicate. So, the middle term 'papers' forming the subject, is not distributed.

Since the middle term is not distributed even once in the premises, no conclusion follows. Hence, the answer is **(e)**.

STATEMENT-ARGUMENTS

In this type of questions, a statement concerned with a political, social or economic issue is given, followed by two arguments, generally one in favour of and one against the statement. The candidate is required to analyse first the statement, then the arguments in context of the statement and decide **which of the arguments holds strong,** and helps, formulate the most appropriate opinion on the subject.

Directions: Each of the following questions consists of a statement followed by two arguments I and II. Give answer:

(a) if only argument I is strong;
(b) if only argument II is strong;
(c) if either I or II is strong;
(d) if neither I nor II is strong and
(e) if both I and II are strong.

Example 1:

Statement: Should number of holidays of government employees be reduced?

Arguments:

I. Yes. Our government employees are having maximum number of holidays among the other countries of the world.

II. Yes. It will lead to increased productivity of government offices.

Sol. Clearly, a comparison with the system in other countries is no strong a criteria for taking a decision on the issue. So, argument I does not hold. Also, reducing the number of holidays implies more working hours which will surely increase productivity.

Hence, the answer is **(b)**.

Example 2:

Statement: Should foreign films be banned in Pakistan?

Arguments:

I. Yes. They depict an alien culture which adversely affects our values.

II. No foreign films are of a high artistic standard.

Sol. Clearly foreign films depict the alien culture but this only helps in learning more. So, argument I does not hold. Also, the reason stated in argument II is not strong enough in contradicting the ban. So, it also does not hold.

Thus, the answer is **(d)**.

Example 3:

Statement: Should there be reservation of seats and posts on ethnic basis?

Arguments:

I. Yes. It will check most of the inter-communal biases.

II. No. Ours is a Islamic state.

Sol. Clearly, reservations on communal basis will increase inter-communal biases. So, argument I is vague. Also, it will be against the Constitution, according to which no communal group is given preference over the others. So, only argument II holds.

Hence, the answer is **(b)**.

EXERCISE

1. **Statement:** Should Pakistan encourage exports, when most things are insufficient for internal use itself?

   Arguments:

   I. Yes. We have to earn foreign exchange to pay for our imports.

   II. No. Even selective encouragement would lead to shortages.

2. **Statement:** Should Pakistan make efforts to harness solar energy to fulfil its energy requirements?

   Arguments:

   I. Yes, Most of the energy sources used at present is exhaustible.

   II. No. Harnessing solar energy requires a lot of capital, which Pakistan lacks in.

3. **Statement:** Should there be students union in college/university?

   Arguments:

   I. No. This will create a political atmosphere in the campus.

   II. Yes, it is very necessary Students are future political leaders.

4. **Statement:** Should cottage industries be encouraged in rural areas?

   Arguments:

   I. Yes. Rural people are creative.

   II. Yes. This would help to solve the problem of unemployment to some extent.

5. **Statement:** Should young entrepreneurs be encouraged?

   Arguments:

   I. Yes. They will help in industrial development of the country.

   II. Yes. They will reduce the burden on employment market.

ANSWERS

1. Clearly, Pakistan can export only the surplus and that which can be saved after fulfilling its own needs, to pay for its imports. Encouragement to export cannot lead to shortages as it shall provide the resources for imports. So, only argument I hold
   Hence, the answer is **(a)**.

2. Clearly, harnessing solar energy will be helpful as it is an inexhaustible resource unlike other resources. So, argument I holds. But argument II is vague as solar energy is the cheapest form of energy. Hence, the answer is **(b)**.
3. The student's union formation shall be a step towards giving to students the basic education in the field of politics. However, it shall create the same political atmosphere in the campus. Thus, both the arguments hold strong. So answer is **(e)**
4. Clearly, cottage industries need to be promoted to create more job opportunities for rural people in the villages themselves. The reason that rural people are creative is vague. So, only argument II holds. Hence, the answer is **(b)**.
5. Clearly, encouraging the young entrepreneurs will open up the field for the establishment of new industries. Thus, it shall help in industrial development and not only employ the entrepreneurs but create more job opportunities for others as well. So, both the arguments hold strong. Hence, the answer is **(e)**.

STATEMENT — ASSUMPTIONS

An **assumption** is something taken for granted, *i.e.* a fact that can be supposed on considering the contents of the given statement. In this type of questions, a statement is given, followed by two assumptions. The candidate is required to assess the given statement and then decide which of the given assumptions is implicit in the statement and choose the same from the alternatives provided.

Directions: In each question below is given a statement followed by two assumptions numbered I and II. Consider the statement and decide which of the given assumptions is implicit.

Give answer

(a) if only assumption I is implicit;
(b) if only assumption II is implicit;
(c) if either I or II is implicit;
(d) if neither I nor II is implicit and
(e) if both I and II are implicit.

Example 1:

Statement: It is desirable to put the child in school at the age of 5 or so.

Assumptions:

I. At that age the child reaches appropriate level of development and is ready to learn.
II. The schools do not admit children after six years of age.

Sol. Since the statement talks of putting the child in school at the age of 5, it means that the child is mentally prepared for the same at this age. So, I is implicit. But, nothing about admission after six years of age is mentioned in the statement. So, II is not implicit.

Hence the answer is **(a)**.

Example 2:

Statement: "You must learn to refer to dictionary if you want to become a good writer."— A advises B.

Assumptions:

I. Only writers refer to the dictionary.
II. All writers good or bad refer to the dictionary.

Sol. It does not follow from the statement that only writers and nobody else refers to the dictionary. Also, nothing is mentioned about bad writers. So, both the assumptions I and II are not implicit. Hence, the answer is **(d).**

Example 3:

Statement: The chairman and secretary of the housing society have requested society members to use water economically to help society to save on water tax.

Assumptions:

I. Majority of members of society are likely to follow the request.
II. It is desirable to reduce expenditure wherever possible.

Sol. Clearly, nothing about the response of society members to the secretary's request can be deduced from the statement. So, I is not implicit. Also, the secretary requests the members to save the money on tax. So, II is implicit. Hence, the answer is **(b).**

EXERCISE

1. **Statement:** "In order to bring punctuality in our office, we must provide conveyance allowance to our employees." - In charge of a company tells Personnel Manager?

Assumptions:

I. Conveyance allowance will not help in bringing punctuality.
II. Discipline and reward should always go hand in hand.

2. **Statement:** "If you trouble me, I will slap you." - A mother warns her child.

Assumptions:

I. With the warning, the child may stop troubling her.
II. All children are basically naughty.

3. **Statement:** "To keep myself up-to-date, I always listen to 9.00 p.m. news on radio."- A candidate tells the interview board.

Assumptions:

I. The candidate does not read newspaper.
II. Recent news is broadcast only on radio.

4. **Statement:** Believe me, I have read it in newspaper X.

Assumptions:

I. Newspaper X gives reliable information/news.

II. I am reporting exactly as it is given in newspaper X.

5. Statement: Highly brilliant and industrious students do not always excel in the written examination.

Assumptions:

I. The written examination is good mainly for mediocre students.

II. The brilliant and industrious students cannot always write good answers in the exam

ANSWERS

1. Assumption I goes against the statement. So, it is not implicit. The allowance will serve as a reward to the employees and shall provoke them to come on time. So, II is implicit. Here Answer is **(b).**
2. The mother warns her child with the expectation that he would stop troubling her. So, I is implicit. The general nature of children cannot be derived from the statement. So, II is not implicit. Here answer is **(a)**
3. The candidate listens to news on the radio does not mean that he does not read newspaper or that radio is the only source of recent news. So, neither I nor II is implicit. Here answer is **(d)**
4. The narrator in the statement clearly insists on the reliability of the fact that what he said, he had read it in newspaper X, and not on the truth of what he said. So, only II is implicit. Here answer is **(b)**
5. Statement II explains the fact given in the statement and so is implicit. Nothing about 'mediocre students' is mentioned in or can be deduced from the given statement. So, I is not implicit. Here answer is **(b).**

STATEMENT — COURSES OF ACTION

A course of action is 'a step or administrative decision to be taken for improvement, follow-up or further action in regard to the problem, policy, *etc.* on the basis of the information given in the statement.'The questions in this section, thus, involve finding the appropriate course of action, assuming the problem or policy being talked about in the statement. In this type of questions, a statement is given followed by two courses of action numbered I and II. The candidate is required to grasp the statement, analyse the problem or policy it mentions and then decide which of the courses of action logically follow.

Example 1:

Many cases of cholera were reported from a nearby village.

I. The question should be raised in the National Assembly.

II. A team of doctors should be rushed to the village.

Sol. Clearly, the disease has to be eradicated. For this, proper and immediate medication and preventive measures by doctors is necessary. So, only course II follows.

Example 2:

People residing in some tribal areas are far from education.

I. Schools for children and adults, should be opened there.

II. Social workers should be entrusted with the job of educating them.

Sol. Clearly, to make permanent arrangements for education in remote tribal areas, schools have to be opened in those very areas. Education by social workers shall be a temporary remedy. So, only the course of action I follows.

EXERCISE

Directions: In each question below is given a statement followed by two courses of action numbered I and II. You have to assume everything in the statement to be true, then decide which of the two suggested courses of action logically follows for pursuing.

Give answer

(a) if only I follows;

(b) if only II follows;

(c) if either I or II follows;

(d) if neither I nor II follows and

(e) if both I and II follow.

1. Statement: A large number of people in ward X of the city are diagnosed to be suffering from a fatal malaria type.

Courses of Action:

I. The city municipal authority should take immediate steps to carry out extensive fumigation in ward X.

II. The people in the area should be advised to take steps to avoid mosquito bites.

2. Statement: Severe drought is reported to have set in several parts of the country.

Courses of Action:

I. Government should immediately make arrangement for providing financial assistance to those affected.

II. Food, water and fodder should immediately be sent to all these areas to save the people and cattle.

3. Statement: A large number of people die every year due to drinking polluted water during the summer.

Courses of Action:

I. The government should make adequate arrangements to provide safe drinking water to all its citizens.

II. The people should be educated about the dangers of drinking polluted water.

4. Statement: Majority of the students in many government schools do not pass in the final examination.

Courses of Action:

I. These schools should be closed down as these have become unproductive.

II. The teachers of these schools should immediately be retrenched.

5. Statement: A very large number of students have failed in the final high school examination due to faulty questions in one of the subjects.

Courses of Action:

I. All the students who have failed in the subject should be allowed to take supplementary examination.

II. All those who are responsible for the error should be suspended and an enquiry should be initiated to find out the facts.

ANSWERS

1. Clearly, prevention from mosquitoes and elimination of mosquitoes are two ways to prevent malaria. So, both the courses follow. Here answer is **(e)**
2. In the break-out of a natural calamity, the basic duty of the government becomes to provide the basic amenities essential to save the lives of people and cattle. Providing financial assistance to all would put undue burden on the country's resources. So, only II follows. Here Answer is **(b).**
3. The situation demands creating awareness among people about the dangers of drinking polluted water so that they themselves refrain from the same, and at the same time taking steps to provide safe drinking water. So, both the courses follow. Here answer is **(e)**
4. Clearly, the situation demands that efforts be made to remove the lackenings in the present system of education and adequate measures be taken to improve the performance of students. Harsh measures as those given in I and II, won't help. So, none of the given courses follows. Here Answer is **(d).**
5. There being faulty questions in the examination paper is a blunder on the part of school management and students should not be made to suffer on account of this. Thus, a re-test should be organised for the students and those responsible for the error be penalised to prevent reoccurrence of such mistake in the future. Hence, both the courses follow. Here answer is **(e)**

STATEMENT — CONCLUSIONS

'Conclusion' means 'a fact that can be truly inferred from the contents of a given sentence or passage.' The questions in this section thus consist of a statement/group of statements, followed by certain inferences based on the facts contained in the given statements. The candidate is required to analyse the given statements, understand their indirect implications and then decide which of the given conclusions follows logically and for sure, from the given statements.

Type 1: Statement-Conclusion

In this type of questions, a statement is given followed by two conclusions. The candidate is required to find out which of these conclusions definitely follows from the given statement and choose the answer accordingly.

Directions: In each of the following questions, a statement is given followed by two conclusions I and II.

Give answer

(a) if only conclusion I follows;
(b) if only conclusion II follows;
(c) if either I or II follows;
(d) if neither I nor II follows and
(e) if both I and II follow.

Example 1.

Statement: Sealed tenders are invited from competent contractors experienced in executing construction jobs.

Conclusions:

I. Tenders are invited only from experienced contractors.
II. It is difficult to find competent tenders in construction jobs.

Sol. According to the statement, tenders are invited from contractors experienced in executing construction jobs. So, conclusion I follow. The availability of competent tenders in construction is not mentioned. So, conclusion II does not follow.

Hence the answer is ***(a)***.

Example 2.

Statement: The manager humiliated Irfan in the presence of his colleagues.

Conclusions:

I. The manager did not like Irfan.
II. Irfan was not popular with his colleagues.

Sol. Clearly, none of the given conclusions is either mentioned in or can be drawn from the facts given in the statement. Hence, the answer is **(d)**.

Example. 3

Statement: Any young man who makes dowry as a condition for marriage discredits himself and dishonours womanhood.

Conclusions:

I. Those who take dowry in marriage should be condemned by society.
II. Those who do not take dowry in marriage respect womanhood.

Sol. Clearly, the statement declares dowry as an evil practice and reflects its demerits. Thus, conclusion I follows. Also, it is given that those who take dowry dishonour womanhood. This implies that those who do not take dowry respect womanhood. So, conclusion II follows. Hence, the answer is **(e)**.

EXERCISE

1. Statement: Morning walks are good for health.

Conclusions:

- **I.** All healthy people go for morning walks.
- **II.** Evening walks are harmful.

2. Statement: Company X has marketed the product. Go ahead, purchase it if price and quality are your considerations.

Conclusions:

- **I.** The product must be good in quality.
- **II.** The price of the product must be reasonable.

3. Statement: The best way to escape from a problem is to solve it.

Conclusions:

- **I.** Your life will be dull if you don't face a problem.
- **II.** To escape from problems, you should always have some solutions with you.

4. Statement: A neurotic is a non-stupid person who behaves stupidly.

Conclusions:

- **I.** Neuroticism and stupidity go hand in hand.
- **II.** Normal persons behave intelligently.

5. Statement: Vegetable prices are soaring in the market.

Conclusions:

- **I.** Vegetables are becoming a rare commodity.
- **II.** People cannot eat vegetables.

ANSWERS

1. **(d) :** The statement mentions that morning walks improve health. But this does not mean that all healthy people go for morning walks. So, I does not follow. Also, nothing is mentioned about evening walks in the statement. So, II does not follow.
2. **(e) :** It is mentioned in the statement that one who considers price and quality before buying a product should buy the product of company X. So, both I and II follow.
3. **(d) :** Clearly, both I and II do not follow from the given statement.
4. **(a) :** It is mentioned in the statement that a neurotic is a person who behaves stupidly. So, I follows. The behaviour of normal persons cannot be deduced from the given statement. So, II does not follow.
5. **(d) :** The availability of vegetables is not mentioned in the given statement. So, I does not follow. Also, II is not directly related to the statement and so it also does not follow.

Type 2: Deriving Conclusions from Passages

In this section of logical deduction, the question consists of a brief passage followed by certain inferences based on it. The candidate is required to analyse the passage and grasp the desirable facts from it. Then, he has to consider each inference in context of the given passage, decide upon its degree of truth or falsity and then choose the best alternative provided accordingly.

Directions: Read the following passage and examine each inference given below it in the context of this passage. Mark your answer as :

(a) if the inference is 'definitely true';

(b) if the inference is 'probably true';

(c) if the 'data provided is inadequate';

(d) if the inference is 'definitely false'; and

Example. 1

The space exploration has been done mainly by using unmanned satellites called space probes containing a large variety of latest scientific instruments on board. These space probes have provided us the close up pictures and other data about planets and other bodies in the outer space. The climax of the intensive American space programme came when Neil Armstrong became the first man to set foot on the moon on July 20, 1969. Originally, the artificial satellites were launched for studying the upper atmosphere of earth.

Statements

1. The space probes have increased our knowledge about space and the bodies in it.
2. Space probes are meant to study the upper atmosphere of earth only.
3. Neil Armstrong was the first man to go into space.
4. Space probes are provided with computers.
5. Moon has been explored by man.

Solution:

1. **(a) :** It is mentioned in the passage that the space probes have provided pictures and certain data of the outer space and the heavenly bodies. Thus, they have helped us increase our knowledge of outer space and the bodies in it.
2. **(e):** According to the passage, the space probes were first designed to study the upper atmosphere of earth. But at present, they are also used to explore outer space and obtain more information about it.
3. **(c):** It is given in the passage that Neil Armstrong was the first man to step foot on moon. But the first man to go into space is not being talked out.
4. **(b):** According to the passage, space probes are provided with large variety of latest scientific instruments. Thus, computers may also be present.
5. **(a):** The fact mentioned in the passage that Neil Armstrong was the first man to set foot on the moon clearly proves that moon has been explored by man.

EXERCISE

Passage. 1

A radical new surgery procedure, laughed at not long ago, is holding out fresh hope for patients of cardiac myopathy, or enlargement of the heart. The technique, now in Pakistan, allows patients to go home two weeks after the operation, to lead a near-normal sedentary life. Cardiac myopathy is a condition that has a variety of causative factors. An attack from one of the 20 identified viruses, parasite infection, long-term alcohol abuse and blood pressure could bring it on, and in rare cases, it could follow child birth and is even known to run in families. The condition is marked by an increase in the size of the heart's chambers and a decrease in the efficiency of pumping.

1. Cardiac myopathy is hereditary.
2. The new technique was never tried in Pakistan in the past.
3. The cardiac myopathy slows down the heartbeat.
4. Earlier the patients suffering from cardiac myopathy were required to travel abroad for such operation.
5. The efficiency of the heart is inversely proportional to the size of the heart.

Solution:

1. **(b):** It is mentioned in the passage that in certain cases, cardiac myopathy was 'known to run in families'. So, it might be possible that it is hereditary.
2. **(a):** The given conclusion can be clearly inferred from the line 'The technique, now in Pakistan , ...' which clearly means that the technique was previously not there in Pakistan .
3. **(a):** It is clearly mentioned in the passage that cardiac myopathy in marked by 'a decrease in the efficiency of pumping'. This means that the heart beat is slowed down.
4. **(c):** Nothing is mentioned about the time before the introduction of the new technique.
5. **(a):** The given inference directly follows from the last line of the passage 'the condition is marked by an increase in the size of the heart's chambers and a decrease in the efficiency of pumping'.

Passage. 2

Though the government cultivates only 3.2 lakh tonnes of mangoes, they are of premium quality and with mangoes becoming second most consumed fruit in the world after grapes, the government has been trying exporting it through sea route which is cheaper. An experiment which was done in this regard last year has proved successful.

1. Quality of mangoes is an important factor in exports.
2. The state also exports good quality grapes.
3. There are some problems in exporting mangoes through sea route.
4. Most of the other exports are through sea routes which is cheaper.
5. The state also cultivates a large number of medium quality of mangoes.

Solution:

1. **(a):** It is given in the passage that mangoes cultivated in the state are of good quality and the government is trying to export them. This implies the given fact.
2. **(c):** Nothing about the production and export of grapes by the state is mentioned in the passage.
3. **(e):** According to the passage, the government is trying to export mangoes through sea route which is cheaper. This clearly means that exporting mangoes through sea route does not entail any problems.
4. **(b):** According to the passage, the government considers sea route a cheaper medium of export. Perhaps the other exports through sea route have given them this experience.
5. **(e):** According to the passage, the state cultivates 3.2 lakh tonnes of mangoes, all of which are of premium quality.

THEME DETECTION

In this type of questions, a paragraph is given followed by certain statements which may or may not be inferred from the passage. The candidate is required to choose that statement which contains the gist or theme of the passage, *i.e.* the idea that it conveys.

Example:

Through advertising, manufacturing exercises a high degree of control over consumer's desires. However, the manufacturer assumes enormous risks in attempting to predict what consumers will want and in producing goods in quantity and distributing them in advance of final selection by the consumers.

The paragraph best supports the statement that manufacturers —

(a) distribute goods directly to the consumers.
(b) can eliminate the risk of overproduction by advertising.
(c) always take moderate and calculated risk.
(d) can predict with great accuracy the success of any product they put on the market.
(e) must depend upon the final consumers for the success of their undertakings.

Solution

According to the passage, it is very difficult for the manufacturer to predict the consumers response to his products. But by advertising, he can stimulate the consumers to buy his product. So, the theme of the paragraph is best mentioned in **(b)**. Hence, **(b)** is the answer.

(a) is incorrect because it is mentioned in the paragraph that manufacturers distribute goods in advance of their demands and not directly to the consumers.
(c) is wrong because according to the passage, manufacturers take 'enormous' and not 'moderate' risks.
(d) is wrong because it is mentioned in the passage that manufacturers take great risk in predicting what the consumers want.
(e) is a true statement but it does not depict the complete theme of the passage.

EXERCISE

Directions: Each of the following questions contains a small paragraph followed by a question on it. Read each paragraph carefully and answer the question given below it:

Passage 1.

The virtue of art does not allow the work to be interfered with or immediately ruled by anything other than itself. It insists that it alone shall touch the work in order to bring it into being. Art requires that nothing shall attain the work except through art itself.

This passage best supports the statement that:

- **(a)** art is governed by external rules and conditions.
- **(b)** art is for the sake of art and life.
- **(c)** art is for the sake of art alone.
- **(d)** artist realises his dreams through his artistic creation.
- **(e)** artist should use his art for the sake of society.

Passage 2.

Though the waste of time or the expenditure on fashions is very large, yet fashions have come to stay. They will not go, come what may. However, what is not required is that strong efforts should be made to displace the excessive craze for fashion from the minds of these youngsters.

The passage best supports the statement that:

- **(a)** fashion is the need of the day.
- **(b)** the excessive craze for fashion is detrimental to one's personality.
- **(c)** the hoard for fashion should be done away with so as not to let down the constructive development.
- **(d)** work and other activities should be valued more than the outward appearance.

Passage 3.

Due to enormous profits involved in smuggling, hundreds of persons have been attracted towards this anti-national activity. Some of them became millionaires overnight. Pakistan has a vast coastline both on the Eastern and Western Coast. It has been a heaven for smugglers who have been carrying on their activities with great impunity. There is no doubt, that from time to time certain seizures were made by the enforcement authorities, during raids and ambush but even allowing these losses the smugglers made huge profits.

The passage best supports the statement that:

- **(a)** smuggling hampers the economic development of a nation.
- **(b)** smuggling ought to be curbed.
- **(c)** authorities are taking strict measures to curb smuggling.
- **(d)** smuggling is fast increasing in our country owing to the quick profit it entails.

Passage 4.

The only true education comes through the stimulation of the child's power by the demands of the social situations in which he finds himself. Through these demands he is stimulated to act as a member of a unity, to emerge from his original narrowness of action and feeling, and to conceive himself from the standpoint of the welfare of the group to which he belongs.

The passage best supports the statement that real education —

(a) will take place if the children imbibe action and feeling.

(b) will take place if the children are physically strong.

(c) is not provided in our schools today.

(d) comes through the interaction with social situations.

(e) comes from the self-centred approach of the students.

Passage 5.

Emerson said that the poet was landlord, sealord, airlord. The flight of imagination made the poet master of land, sea and air. But a poet's dream of yesterday becomes today an actual achievement and a reality for all men. Even those who invented, improved and perfected the aeroplane could hardly have dreamt of the possibility of flight into outer space.

The passage best supports the statement that:

(a) seemingly impossible imaginations make one a good poet.

(b) all imaginations become a reality some day.

(c) what man imagined has never been impossible; he has always turned it a reality through his conception of ideas and sheer hard labour.

(d) man has reached the climax of technological development with his exploration into outer space.

ANSWERS

Passage 1 **(c)** Passage 2 **(c)** Passage 3 **(d)** Passage 4 **(d)** Passage 5 **(c)**

CHAPTER 19

BASIC MATHEMATICS

1. NUMBER SYSTEM

"NUMBER RULES THE UNIVERSE." PYTHAGORAS

Natural Numbers

These are also called *Counting numbers* 1, 2, 3, 4, 5, ... and are known as natural numbers.

The set of all natural numbers can be represented by N.

$$N = \{1, 2, 3, 4, 5, \ldots\}$$

Whole Numbers

If we include 0 among the natural numbers, then the numbers 0, 1, 2, 3, 4, 5, ... are called whole numbers. The set of whole numbers can be represented by w. Clearly, every natural number is a whole number but 0 is a whole number which is not a natural number.

$$W = \{0, 1, 2, 3, 4, 5, \ldots\}$$

Integers

All counting numbers and their negatives including zero are known as integers. Z or I.

The set of integers can be represented by

$$Z \text{ or } I = \{\ldots -4, -3, -2, -1, 0, 1, 2, 3, 4, \ldots\}$$

- **Positive Integers**: The set $I+ = \{1, 2, 3, 4, \ldots.\}$ is the set of all positive integers.
- **Negative Integers**: The set $I- = \{-1, -2, -3, \ldots\}$ is the set of all negative integers. 0 is neither positive nor negative.
- **Non-negative Integers**: The set $\{0, 1, 2, 3, \ldots\}$ is the set of all non-negative integers.

Note: Only whole numbers are integers. Therefore,: fractions, decimals, percent and exponents can never be integers.

Rational Numbers

The numbers of the form p/q, where p and q are integers and q ≠ 0, are known as rational numbers, e.g. $\frac{4}{7}, \frac{3}{2}, -\frac{5}{8}, \frac{0}{1}, -\frac{2}{3},$ etc. The set of all rational numbers is denoted by Q.

Irrational Numbers

Those numbers which when expressed in decimal form are neither terminating nor repeating decimals are known as irrational numbers, e.g. $\sqrt{2}, \sqrt{3}, \sqrt{5}, \pi$, etc. Any number that is not rational is called an irrational number. Irrational cannot be expressed in the form of fraction.

$1.5 = \frac{3}{2}$ > Ratio $\pi = 3.14159 \ldots = \frac{?}{?}$ (No Ratio)

Rational Irrational

Example: π (pi) is a famous irrational number.

π = 3.1415926535897932384626433832795 (and more...). You cannot write down a simple fraction that equals Pi. The popular approximation of 22/7 = 3.1428571428571... is close but not accurate. Another clue is that the decimal goes on forever without repeating. The square root of 2 is an irrational number, meaning its decimal equivalent goes on forever, with no repeating pattern:

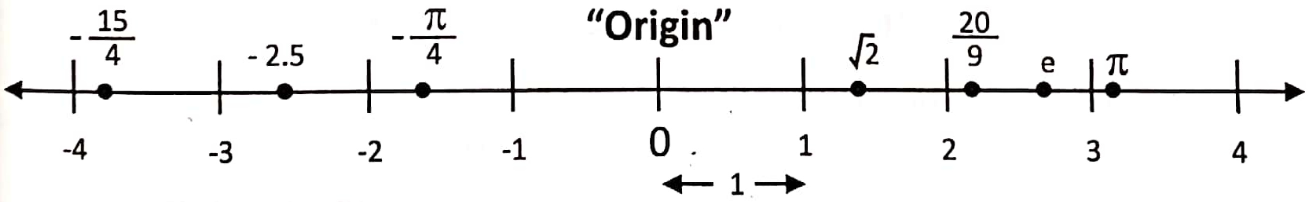
$$\sqrt{2} = 1.41421356237309\ldots$$

Real Numbers

The real numbers is the set of numbers containing all of the rational numbers and all of the irrational numbers. The real numbers are "all the numbers" on the number line. There are infinitely many real numbers just as there are infinitely many numbers in each of the other sets of numbers. Nearly any number you can think of is a real number. Real numbers can also be positive, negative or zero.

Number line

A straight line on which each point represents a real number. It is a geometric representation of numerical values. A point is chosen on the line to be the "**origin**", points to the right are positive, and points to the left are negative.



A distance is chosen to be "1", then whole numbers are marked off: {1,2,3,...}, and also in the negative direction: {–1,–2,–3, ...}.

Even and Odd Numbers

Even numbers can be divided evenly into groups of two. The number four can be divided into two groups of two. Even numbers always end with a digit of 0, 2, 4, 6 or 8.

2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30… and so on, are even numbers.

Odd numbers can NOT be divided evenly into groups of two. The number five can be divided into two groups of two and one group of one. Odd numbers always end with a digit of 1, 3, 5, 7, or 9.

1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31... are odd numbers.

Consider the number 3,842,917. It ends in 7, an odd number. Therefore, 3,842,917 is an odd number. Likewise, 8,322 is an even number because it ends in 2.

Remember: Zero is an even number because it satisfies the condition of even number, it is an integer multiple of 2.

Prime Numbers

A prime number can be divided, without a remainder, only by itself and by 1. For example, 17 can be divided only by 17 and by 1.

Some facts:

- The only even prime number is 2. All other even numbers can be divided by 2 also.
- No prime number greater than 5 ends in a 5. Any number greater than 5 that ends in a 5 can be divided by 5.
- Zero and 1 are not considered prime numbers.

Here is a table of all prime numbers up to 200:

	2	3	5	7	11	13	17	19	23
29	31	37	41	43	47	53	59	61	67
71	73	79	83	89	97	101	103	107	109
113	127	131	137	139	149	151	157	163	167
173	179	181	191	193	197	199			

Composite Numbers

Natural numbers greater than 1 which are not prime, are known as composite numbers. For example, each of the numbers 4, 6, 8, 9, 12, etc., are composite numbers.

MULTIPLE CHOICE QUESTIONS

1. **Which number is not rational?**
   (a) 4 (b) 0.3 (c) 6.171717 (d) $\sqrt{3}$
2. **Which number is rational, an integer, a whole number, and a natural number?**
   (a) 0.17 (b) 36/12 (c) 0 (d) 3/4
3. **When is a fraction irrational?**
   (a) When it is a mixed number (b) When it is improper
   (c) When it is negative (d) Never
4. **The number √16 is which of these following?**
   (a) Rational (b) Integer (c) Whole (d) All of these
5. **The number 1.34343434... is which of the following?**
   (a) Irrational (b) Rational
   (c) Rational and Integer (d) Rational, Integer, and Whole
6. **π (pi) is which of the following?**

(a) Irrational (b) Irrational and Integer
(c) Rational (d) Rational and Integer

7. **Which number is the only whole number that is not a natural number?**
(a) -1 (b) 1 (c) 0 (d) 1/2

8. **Which number is a non-terminating/non-repeating decimal?**
(a) √144 (b) 1.626262... (c) 9.089 (d) √5

9. **The symbol used to represent the set of integers is _____.**
(a) R (b) W (c) Q (d) Z or I

10. **Which number is not an integer?**
(a) √81 (b) 4.3333... (c) -2 (d) 0

ANSWERS

1.	d	2.	b	3.	d	4.	d	5.	b	6.	a	7.	c
8.	d	9.	d	10.	b								

2-HIGHEST COMMON FACTOR (HCF) AND LEAST COMMON MULTIPLE (LCM)

Factor is defined as a number that divides exactly into another number and which indicates a specific multiple, number or quantity. It can also be called as a **divisor** of the number. So, 1 is a factor of all existing numbers because 1 can be multiplied with any number.

Example: 1 x 2 x 3 = 6

Therefore, 1, 2 and 3 are the factors of the number 6.

Finding Factors of a Number

There are two ways to find out factors of a number.

- Factoring through division method.
- Factoring through tree method.

Example: Find the factors of 16 using division method:

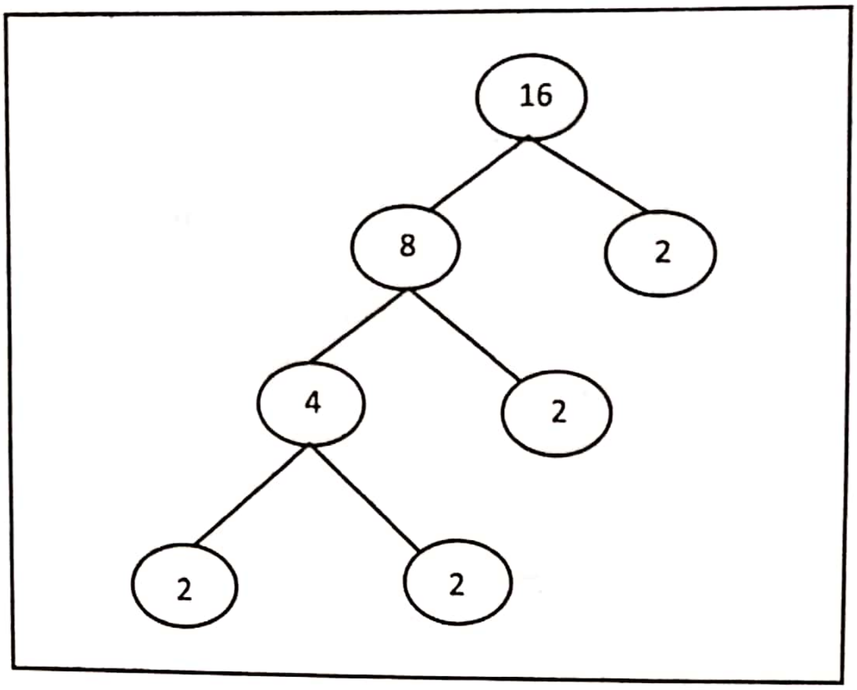
- **Step 1:**

$$\begin{array}{r|l} 2 & 16 \\ \hline 2 & 8 \\ \hline 2 & 4 \\ \hline 2 & 2 \\ \hline & 1 \end{array}$$

- **Step 2:** If we divide 16 by the prime factor until we have 1 as a quotient.

Therefore, the factors of 16 are, 2 x 2 x 2 x 2

Find the factors of 16 using tree method



In the above diagram the starting of a node is rooting of a tree, it is 16. That is the parent node of 16 divided into two child nodes, 8 and 2. Here, the node have not a child node while 8 is divided again 2. This process is repeated until, we get the prime number in the child node.

The product of the node without a child provide the factors of 16, such as 2 x 2 x 2 x 2

Highest Common Factor (H.C.F).

The **Highest Common Factor (H.C.F.)** of two (or more) numbers is the largest number that divides evenly into both numbers. In other words, the H.C.F. is the largest of all the common factors. The other names for HCF are **Greatest Common Divisor (GCD)**, and Greatest Common Denominator. It is very easy to find a H.C.F. of small numbers, like 6 and 9 (it is 3) or 8 and 4 (it is 4). The best way is to keep finding the factors of the smaller number, starting from the largest factor. The first factor of the smaller number that is also a factor of the larger number is a H.C.F.

Finding the H.C.F.

For larger numbers one can use the following method:

1. Find all prime factors of both numbers.
2. Write both numbers as a multiplication of prime numbers.
3. Find which factors are repeating in both numbers and multiply them to get H.C.F.

Example: Find prime factors of **24** and **36**:

$$24 = 2 \times 2 \times 2 \times 3$$
$$36 = 2 \times 2 \times 3 \times 3$$

If we write down the numbers that are the same in both lists, they will give us the highest common factor of 24 and 36:

HCF of 24 and 36 is 2 x 2 x 3 = 12

Example: Find the Highest Common Factor (H.C.F.) of **240** and **924.**

We will start with the smallest prime number and we will divide 240 into it if we can. We will do the same with the result (or quotient), and we will keep dividing by prime numbers until we have 1 as a quotient. Each time we write the prime factor to the right and the quotient below:

240	2	2 is a factor of 240;
120	2	2 is a factor of 120;
60	2	2 is a factor of 60;
30	2	2 is a factor of 30;
15	3	3 is a factor of 15;
5	5	5 is a factor of 5;
1		

240 = 2 x 2 x 2 x 2 x 3 x 5

Finding all prime factors of 924:

924	2	2 is a factor of 924;
462	2	2 is a factor of 462;
231	3	3 is a factor of 231;
77	7	7 is a factor of 77;
11	11	11 is a factor of 11;
1		

924 = 2 x 2 x 3 x 7 x 11

Multiply the factors which repeat in both numbers to get the H.C.F.

The Highest Common Factor is 2 x 2 x 3 = 12

Least Common Multiple (LCM)

The least common multiple is the smallest positive number that is a multiple of two or more numbers. In mathematics, LCM is referred as lowest common multiple, least common multiple, smallest common multiple or least common factor. A **multiple** of a number is the product of that number and any other whole number. Multiples are what we get after multiplying the number by an integer (not a fraction). Example: Multiples of 3: 3, 6, 9, 12, …

So we know that 9 is a multiple of 3, as 3 × 3 = 9

But 7 is NOT a multiple of 3

How to Find the Least Common Multiple

There are two widely used methods to find the LCM.

Method 1

- Step 1 - We write the multiples of each number (multiplying by 1, 2, 3, etc.).
- Step 2 - Then we look for the smallest number that appears in each list.

Example: Least Common Multiple of 3 and 5:

- List the Multiples of each number,

- The multiples of 3 are 3, 6, 9, 12, 15, 18, ... etc
- The multiples of 5 are 5, 10, 15, 20, 25, ... etc
- Find the first Common (same) value:

15 is a common multiple of 3 and 5, and is the smallest, or least, common multiple

Example: consider two numbers 4 and 6.

- Multiples of 4 are: 4, 8, 12, 16, 20, 24, 28, 32, 36....
- Multiples of 6 are: 6, 12, 18, 24, 30, 36, 42, 48......

We observe that the common multiples are: 12, 24, 36.....

So the least common multiple is 12.

It is simply the smallest of the common multiples.

Method 2 — Prime Factorization Method

To use this method factor each of the numbers into primes. Then, for each different prime number in all of the factorizations, we do the following steps:

- **Step 1:** Count the number of times each prime number appears in each of the factorizations.
- **Step 2:** For each and every prime number, take the biggest number of these counts.
- **Step 3:** The least common multiple is the multiple of all the prime numbers having highest power.

Example: Find the least common multiple of 24, 36 and 40.

$$\text{We have, } 24 = 2\text{x}2\text{x}2\text{x}3 = 2^3 \times 3$$
$$36 = 2\text{x}2\text{x}3\text{x}3 = 2^2 \times 3^2$$
$$40 = 2\text{x}2\text{x}2\text{x}5 = 2^3 \times 5$$

Here, 2, 3 and 5 are the prime factors. Highest power of 2, 3 and 5 are 3, 2 and 1, respectively.

$$\text{Hence, the LCM of 24, 36 and 40} = 2^3 \times 3^2 \times 5 = 8 \times 9 \times 5 = 360.$$

SELF-EXERCISES

Use Method number 1 to find the LCM:

1. 15 and 12
2. 14 and 28
3. 18 and 12
4. 10, 12, and 8
5. 14, 21 and 35
6. 30, 50 and 60

Use Method number 2 (Prime Factors) to find the LCM:

7. 24 and 20
8. 9, 15 and 25

Find the GCF and LCM:

9. 24, 32
10. 72, 40
11. 98, 35
12. 490, 700

13 It takes Jun Kit 35 seconds to pack and label a box of electronic components. The same job takes Lucas 42 seconds and Fleming 28 seconds. If they all start at the same time, after how many seconds will they all want to use the labelling machine at once?

14. Spring flowering bulbs are on sale at Austin's garden shop. Travis and Sydney want to buy equal amounts of all four kinds. What is the least number of each kind they can they can buy? (Crocus – 6 for \$1.65; Tulip – 4 for \$3.50; Daffodil – 3 for \$1.95; Hyacinth – 5 for \$2.25).

15. Brendan and Paul work part-time in the local pet store. Brendan works every fourth day. Paul works every sixth day. The store is open seven days a week. When will they work together?

3-EXPONENTS

The **exponent** of a number says **how many times** to use the number in a multiplication. In 8^2 the "2" says to use 8 twice in a multiplication, so

$$8^2 = 8 \times 8 = 64$$

In other words: 8^2 could be called "8 to the power 2" or "8 to the second power", or simply "8 squared". Exponents are also called *Powers or Indices*. The thing that's being multiplied, being 5 in this example, is called the "base" and 2 is the "power".

- **Example:** $5^3 = 5 \times 5 \times 5 = 125$
  In words: 5^3 could be called "5 to the third power", "5 to the power 3" or simply "5 cubed".

- **Example:** $2^4 = 2 \times 2 \times 2 \times 2 = 16$
  In words: 2^4 could be called "2 to the fourth power" or "2 to the power 4" or simply "2 to the 4th". Exponents make it easier to write and use many multiplications.

Negative Exponents

Negative? What could be the opposite of multiplying? Dividing! A negative exponent means how many times **to divide** one by the number. Example: $8^{-1} = 1 \div 8 = 0.125$

Examples:

Negative Exponent	Reciprocal of Positive Exponent	Answer
4^{-2}	$= 1 / 4^2$	$= 1/16 = 0.0625$
10^{-3}	$= 1 / 10^3$	$= 1/1{,}000 = 0.001$
$(-2)^{-3}$	$= 1 / (-2)^3$	$= 1/(-8) = -0.125$

Fractional Exponents

- An exponent of $^1/_2$ is actually **square root for example** $2^{\frac{1}{2}}$ **is equal to** $\sqrt{2}$
- And an exponent of $^1/_3$ is **cube root. Example** $2^{\frac{1}{3}}$ **is the same as** $\sqrt[3]{2}$
- An exponent of $^1/_4$ is **4th root**

Example: What is $9^{\frac{1}{2}} \times 9^{\frac{1}{2}}$?

$$9^{\frac{1}{2}} \times 9^{\frac{1}{2}} = 9^{\left(\frac{1}{2}+\frac{1}{2}\right)} = 9^{(1)} = 9$$

So $9^{\frac{1}{2}}$ times itself gives 9.

Rules for Exponents

Below is List of Rules for Exponents and an example of using each rule:

Zero-Exponent Rule: $a^0 = 1$, this says that anything raised to the zero power is 1.	$3^0 = 1$ $(5x^3y^4)^0 = 1$
Power Rule (Powers to Powers): $(a^m)^n = a^{mn}$, this says that to raise a power to a power you need to multiply the exponents.	$(x^5)^4 = x^{20}$ $(2x^4y^2)^3 = 2^3x^{12}y^6 = 8x^{12}y^6$
Negative Exponent Rule: $a^{-n} = \frac{1}{a^n}$, this says that negative exponents in the numerator get moved to the denominator and become positive exponents.	$5^{-2} = \frac{1}{5^2} = \frac{1}{25}$ $\frac{x^{-3}}{y^{-7}} = \frac{y^7}{x^3}$
Product Rule: $a^m \cdot a^n = a^{m+n}$, this says that to multiply two exponents with the *same base*, you keep the base and add the powers.	$x \cdot x^5 = x^6$ $y^4 \cdot y^9 = y^{13}$
Quotient Rule: $\frac{a^m}{a^n} = a^{m-n}$, this says that to divide two exponents with the *same base*, you keep the base and subtract the powers.	$\frac{x^5}{x^3} = x^{5\text{-}3} = x^2$ $\frac{y^4}{y^9} = \frac{1}{y^5}$

We have reviewed the rules for exponents, here are the steps required for simplifying exponential expressions (notice that we apply the rules in the same order the rule were written above):

Example: Simplify: $\left(\frac{3x^3y^{-2}}{4x^5y^{-3}}\right)^{-3}$

Step 1: Apply the Zero-Exponent Rule. In this case, there are no zero powers.	$\left(\frac{3x^3y^{-2}}{4x^5y^{-3}}\right)^{-3}$
Step 2: Apply the Power Rule.	$\frac{3^{-3}x^{-9}y^6}{4^{-3}x^{-15}y^9}$
Step 3: Apply the Negative Exponent Rule. Move every negative exponent in the numerator to the denominator and vice versa.	$\frac{y^6 4^3 x^{15}}{3^3 x^9 y^9}$
Step 4: Apply the Product Rule. In this case, the product rule does not apply.	$\frac{y^6 4^3 x^{15}}{3^3 x^9 y^9}$

Step 5: Apply the Quotient Rule. In this case, the x's ended up in the numerator and the y's ended up in the denominator.	$\frac{4^3 x^6}{3^3 y^3}$
Step 6: Raise each coefficient (or number) to the appropriate power and then simplify or reduce any remaining fractions. In this case, the fraction does not reduce.	$\frac{64x^6}{27y^3}$

MULTIPLE CHOICE QUESTIONS (MCQS)

1. **Write the base of $-(-6)^5$.**
   (a) 6 (b) -6 (c) -6×5 (d) 5
2. **Evaluate: 4^6**
   (a) 1296 (b) 4096 (c) 10 (d) 24
3. **Write one billion as a power of 10.**
   (a) 10^8 (b) 10^9 (c) 10^{10} (d) 10^{12}
4. **Evaluate: $(-15)^0$**
   (a) -1 (b) 1 (c) 0 (d) -15
5. **Evaluate: $-(10^0)^7$**
   (a) -7 (b) 1 (c) 7 (d) -1
6. **State which operation you would do first to evaluate $8 + 9 \times 6^2 - 5$.**
   (a) Square 6 (c) Subtract 5 from 6
   (b) Add 8 and 9 (d) Multiply 9 & 6
7. **Evaluate: $5^3 - (-6)^3$**
   (a) 33 (b) -91 (c) 341 (d) -3
8. **Evaluate: $(2 + 3)^2 - (3 - 5)^3$**
   (a) 17 (b) -85 (c) 16 (d) 33
9. **Write the product of $5^2 \times 5^4$ as a single power.**
   (a) 10^6 (b) 5^6 (c) 25^6 (d) 5^8
10. **Which answer is negative? i) $(-7)^{10}$, ii) $-(7)^{10}$, iii) $-(-7)^{10}$**
    (a) i only (b) i and iii (c) i and ii (d) ii and iii

ANSWERS

1.	b	2.	b	3.	b	4.	b	5.	d	6.	a	7.	c
8.	d	9.	b	10.	d								

4-SQUARE ROOTS AND CUBE ROOTS

"Roots" (or "radicals") are the "opposite" operation of applying exponents; you can "undo" a power with a radical, and a radical can "undo" a power. For instance, if you square 2, you get 4, and if you "take the square root of 4", you get 2.

$$2^2 = 4, \text{ so } \sqrt{4} = 2$$

$$3^2 = 9, \text{ so } \sqrt{9} = 3$$

The "$\sqrt{\ }$" is called the "radical" symbol. The expression "$\sqrt{9}$" is read as "root nine", "radical nine", or "the square root of nine".

You can raise numbers to powers other than just 2; you can cube things, raise them to the fourth power, raise them to the 100th power, and so forth. In the same way, you can take the cube root of a number, the fourth root, the 100th root, and so on.

To indicate some root other than a square root, you use the same radical symbol, but you insert a number into the radical, tucking it into the "check mark" part. For instance, $4^3 = 64$, so $\sqrt[3]{64} = 4$

The "3" in the above is the "index" of the radical; the "64" is "the argument of the radical", also called "the radicand".

- a square (second) root is written as $\sqrt{\ }$
- a cube (third) root is written as $\sqrt[3]{\ }$
- a fourth root is written as $\sqrt[4]{\ }$
- a fifth root is written as $\sqrt[5]{\ }$

Simplifying Square Root Terms

To simplify a square root, you "take out" anything that is a "perfect square"; that is, you take out front anything that has two copies of the same factor:

- $\sqrt{4} = \sqrt{2^2} = 2$
- $\sqrt{49} = \sqrt{7^2} = 7$
- $\sqrt{225} = \sqrt{15^2} = 15$

Note that the value of the simplified radical is always *positive*. When you solve the equation $x^2 = 4$, you are trying to find *all* possible values that might have been squared to get 4. But when you are just simplifying the expression $\sqrt{4}$, the ONLY answer is "2"; this positive result is called the "principal" root.

Simplify $\sqrt{144}$

There are various ways. You can approach this simplification. One would be by factoring and then taking two different square roots:

$$\sqrt{144} = \sqrt{9 \times 16} = \sqrt{9}\sqrt{16} = 3 \times 4 = 12$$

The square root of 144 is 12.

You probably already knew that $12^2 = 144$, so obviously the square root of 144 must be 12.

Simplify $\sqrt{24}\sqrt{6}$

Neither of 24 and 6 is a square, but what happens if you multiply them inside one radical?

$$\sqrt{24}\sqrt{6} = \sqrt{24\times 6} = \sqrt{144} = \sqrt{12\times 12} = 12$$

Simplify $\sqrt{75}$

$$\sqrt{75} = \sqrt{3\times 25} = \sqrt{25}\sqrt{3} = 5.\sqrt{3}$$

This answer is pronounced as "five root three". It is proper form to put the radical at the end of the expression. While $\sqrt{3}$ cannot be simplified further.

Simplify $\sqrt{20r^{18}st^{21}}$

The 20 factors as 4×5, with the 4 being a perfect square. The r^{18} has nine pairs of r's; the s is unpaired; and the t^{21} has ten pairs of t's, with one t left over. Then:

$$\sqrt{20r^{18}st^{21}} = \sqrt{4\times r^{18}\times t^{20}\times 5\times s\times t}$$

$$= 2r^{9}t^{10}\sqrt{5st}$$

Perfect Square Factors

A number is known as a perfect square if it can be expressed as the product of any other whole number in itself twice. For example, 25 is a perfect square as it can be expressed as 5 × 5. When evaluating (often referred to as taking) the square root of a given number, find if it has any perfect square factors or not. The following is the list of perfect squares from 1 to 100.

$$\sqrt{1} = 1 \text{ since } 1^2 = 1$$

$$\sqrt{4} = 2 \text{ since } 2^2 = 4$$

$$\sqrt{9} = 3 \text{ since } 3^2 = 9$$

$$\sqrt{16} = 4 \text{ since } 4^2 = 16$$

$$\sqrt{25} = 5 \text{ since } 5^2 = 25$$

$$\sqrt{36} = 6 \text{ since } 6^2 = 36$$

$$\sqrt{49} = 7 \text{ since } 7^2 = 49$$

$$\sqrt{81} = 9 \text{ since } 9^2 = 81$$

$$\sqrt{100} = 10 \text{ since } 10^2 = 100$$

The **Perfect Cubes** are the cubes of the whole numbers:

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	etc
Perfect Cubes:	1	8	27	64	125	216	343	512	729	1000	1331	1728	2197	2744	3375	...

It is easy to work out the cube root of a perfect cube, but it is **really hard** to work out other cube roots.

Properties of Square Roots

Properties of square roots and radicals guide us on how to deal with roots when they appear in algebra. Other properties are

- $\sqrt{x} \times \sqrt{y} = \sqrt{xy}$
- $\sqrt{\frac{x}{y}} = \frac{\sqrt{x}}{\sqrt{y}}$
- $\sqrt{x+y} \neq \sqrt{x} + \sqrt{y}$
- $a\sqrt{x} + b\sqrt{x} = (a+b)\sqrt{x}$
- $a\sqrt{x} - b\sqrt{x} = (a+b)\sqrt{x}$
- $a\sqrt{x} = \sqrt{a^2 \times x}$

MULTIPLE CHOICE QUESTIONS (MCQs)

Note: Do not use calculator while solving those questions.

1. **Which of the following is not a square number?**

(a) 4 **(b)** 9 **(c)** 16 **(d)** 24

2. **The square of 23 is :**

(a) 529 **(b)** 526 **(c)** 46 **(d)** 429

3. **The square of which of the following would be even number?**

(a) 2826 **(b)** 7779 **(c)** 1057 **(d)** 131

4. **How many natural numbers lie between 9^2 and 10^2?**

(a) 15 **(b)** 19 **(c)** 18 **(d)** 17

5. **The square of -9 is**

(a) -81 **(b)** 81 **(c)** 18 **(d)** -18

6. **What is the smallest square number which is divisible by each of the numbers 6,9 and 15 ?**

(a) 900 **(b)** 810 **(c)** 630 **(d)** 720

7. **The cube of an even number is always ____________.**

(a) odd number **(b)** even number **(c)** prime number **(d)** none of these

8. **The symbol for cube root is __________.**

(a) $\sqrt{3}$ **(b)** $\sqrt[3]{\ }$ **(c)** $\sqrt{3}$ **(d)** $\sqrt[2]{3}$

9. **How many zeros will be there in the cube root of 27000?**

(a) 3 **(b)** 0 **(c)** 1 **(d)** 2

10. If $8^3=512$, then $\sqrt[3]{512}$ = __________.

(a) 8 (b) 7 (c) 13 (d) 9

ANSWERS

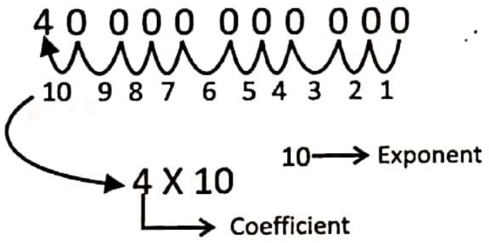
1.	d	2.	a	3.	a	4.	c	5.	b	6.	a	7.	b
8.	b	9.	c	10.	a								

5-SCIENTIFIC NOTATION

Mathematicians have developed a shorter method to express very large numbers. This method is called **scientific notation.** Scientific Notation is based on powers of the base number 10. The number 123,000,000,000 in scientific notation is written as:

$$1.23 \times 10^{11}$$

The first number 1.23 is called the coefficient or the digit term. It must be greater than or equal to 1 and less than 10. *The second number is called the base.* It must always be 10 in scientific notation. The base number 10 is always written in exponent form. In the number 1.23 x 10^{11} the number 11 is referred to as the exponent or power of ten. See example below



How to write a number in scientific notation? Example 123,000,000,000

Put the decimal after the first digit and drop the zeroes.

1.23000000000

1.23

In the number 123,000,000,000 the coefficient will be 1.23.

To find the exponent count the number of places from the decimal to the end of the number. In 123,000,000,000 there are 11 places. Therefore, we write 123,000,000,000 as:

$$1.23 \text{ X } 10^{11}$$

The number 123,000,000,000 can also be written as: 1.23E+11 or as 1.23 X 10^11

Here are some examples of scientific notation.

Examples	
$10000 = 1 \times 10^4$	$24327 = 2.4327 \times 10^4$
$1000 = 1 \times 10^3$	$7354 = 7.354 \times 10^3$
$100 = 1 \times 10^2$	$482 = 4.82 \times 10^2$
$10 = 1 \times 10^1$	$89 = 8.9 \times 10^1$ (not usually done)

$1 = 10^0$	$1 = 5000^0$
$1/10 = 0.1 = 1 \times 10^{-1}$	$0.32 = 3.2 \times 10^{-1}$ (not usually done)
$1/100 = 0.01 = 1 \times 10^{-2}$	$0.053 = 5.3 \times 10^{-2}$
$1/1000 = 0.001 = 1 \times 10^{-3}$	$0.0078 = 7.8 \times 10^{-3}$
$1/10000 = 0.0001 = 1 \times 10^{-4}$	$0.00044 = 4.4 \times 10^{-4}$

As you can see, the exponent of 10 is the number of places the decimal point must be shifted to give the number in long form.

A **positive** exponent shows that the decimal point is shifted that number of places to the right for example $24327 = 2.4327 \times 10^4$. A **negative** exponent shows that the decimal point is shifted that number of places to the left for example $0.00044 = 4.4 \times 10^{-4}$.

6-ROUNDING NUMBERS

Rounding is used to make a number easier to work with. For example, if you know that there are 496 students in your college, you can say that there are approximately 500 students in your college. When a number is rounded (or rounded off), it is approximated by eliminating the least significant digits. For example, the number 42 can be rounded down to 40 (this number was rounded to the tens place). Similarly, 285 can be rounded up to 300 (this number was rounded to the hundreds place). See examples given in the diagram below.

①2 → 10
1①4 → 110
⑤7 → 60
1,3③4 → 1330
1,4⑧8 → 1490
⑨7 → 100

The word significant means: having meaning.

With the number 368249, the 3 is the most significant digit, because it tells us that the number is 3 hundred thousand and something. It follows that the 6 is the next most significant, and so on. With the number 0.0000058763, the 5 is the most significant digit, because it tells us that the number is 5 millionths and something. The 8 is the next most significant, and so on.

Whole numbers can be rounded to the tens place, hundreds place, thousands place, and so on. Decimal numbers can also be rounded; this approximates the number to the nearest tenth, hundredth, thousandth, or other decimal place.

How to Round Off a Number

Example: Round of 246, 257 to tens place

1. **Determine which digit is to be rounded** - now look at the digit to the right of this. For example, if you are rounding the number 246 to the tens digit, the digit "4" (the tens digit) will be rounded and the "6" (the ones digit) is the digit that will become zero.

2. **If the digit to become zero is 0, 1, 2, 3, or 4,** then the digit to be rounded (the digit immediately to the left of it) remains the same.
3. **If the digit to become zero is 6, 7, 8 or 9,** then the digit to be rounded is increased by one. In our example of rounding 246 to the tens digit, the "6" is large enough to make the next digit increase by one (from "4" to "5"), and the number is rounded to 250.
4. **If the digit to become zero is 5:** If the digit to the right of the digit is to be rounded is a five (like 15 rounded to the tens digit or 257 rounded to the hundreds digit), then there are two possibilities.

4A. **If there is at least one non-zero digit after the five,** then the digit to be rounded increases by one and the five becomes a zero (as do all the remaining digits). For example, 257 rounded to the tens digit becomes 260, and 1521 rounded to the thousands place becomes 2000.

4B. **If there are no digits after the five** (or the digits after the five are all zeros), then the number could equally well be rounded up or down. Mathematicians have devised a rule so that in large groups of data, half the time 5s are rounded up and half the time 5s are rounded down, producing no bias in the data.

Examples of Rounding Numbers

Number	Nearest ten	Nearest hundred	Nearest thousand
1,234	1,230	1,200	1,000
89,355	89,360	89,400	89,000

Decimal numbers can also be rounded, estimating the number to the nearest tenth, hundredth, thousandth, and so on.

Examples of Rounding Decimals

Number	Nearest tenth	Nearest hundredth	Nearest thousandth
1.2346	1.2 or 1.3	1.23	1.235
31.6479	31.6	31.65	31.648

EXERCISE

1. **Round off each of the following to the nearest whole number.**
   (a) 8.71 (b) 26.01 (c) 69.48 (d) 103.72
2. **Round off the decimal:**
   (a) 3.153 to the nearest tenths
   (b) 10.126 to the nearest hundredths
   (c) 5.0014 to the nearest thousandths
   (d) 5.992 to the nearest tenths
3. **Find the value of**
   (a) 23.37 correct to 1 decimal place
   (b) 1.2003 correct to 3 decimal places
   (c) 7.755 correct to 2 decimal places
   (d) 9.99 correct to 1 decimal place
4. **Find the number of significant digits in the following.**
   (a) 5.60 (b) 0.09 (c) 80.80 (d) 9.03

ANSWERS

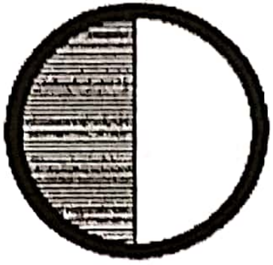
1.	**(a)**	9	**(b)**	26	**(c)**	69	**(d)**	104
2.	**(a)**	3.2	**(b)**	10.13	**(c)**	5.001	**(d)**	6
3.	**(a)**	23.4	**(b)**	1.200	**(c)**	7.76	**(d)**	10.0
4.	**(a)**	3	**(b)**	1	**(c)**	4	**(d)**	3

CHAPTER 20

FRACTIONS AND DECIMAL

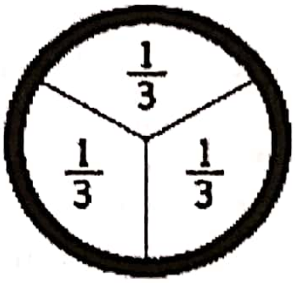
FRACTIONS

If you cut an object into smaller parts you need a way of expressing what these parts are. For example, you can cut a pie into two equal pieces so that there are **two halves**. The two halves make up the whole pie. You can write this mathematically as:



$$\frac{1}{2}+\frac{1}{2}=1$$

You can also cut the pie into three equal pieces, i.e. thirds. The three thirds make up the whole pie. You can write another mathematical expression for this, too.



$$\frac{1}{3}+\frac{1}{3}+\frac{1}{3}=1$$

This way of expressing a part of a whole mathematically is called a **fraction**.

The top number (**numerator**) of a fraction tells how many parts there are. The bottom number (**denominator**) of a fraction tells how many parts make a whole.

$$\frac{1}{2} \Rightarrow \frac{\text{Numerator}}{\text{Denominator}}$$

Every fraction tells us about itself and the original whole it came from. For example, 2/5 means there were 5 parts in the whole and you have 2 of them.

Types of Fractions

There are several types of fractions.

Common fractions (also called **vulgar fractions** or **proper fractions**). This type of fraction is smaller than one but greater than zero. Denominator is larger than numerator.

$$\frac{1}{5} \text{ or } \frac{2}{3}$$

Improper fractions are fractions where the top number (numerator) is larger than the bottom number (denominator). This means that this type of fraction is actually greater than one.

$$\frac{5}{4} \text{ or } \frac{3}{2}$$

Mixed numbers are those where a fraction is written after a whole number.

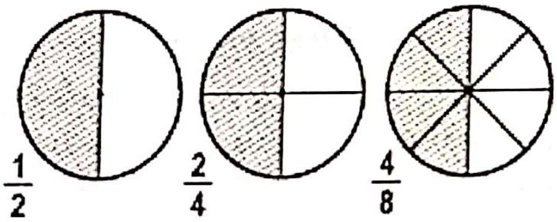
$$1\frac{1}{4}$$

An improper fraction can be rearranged to be written as a mixed number.

$$\frac{6}{5} = 1\frac{1}{5}$$

Equivalent fractions are fractions that have the same value as each other.

$$\frac{1}{2} = \frac{2}{4} = \frac{4}{8}$$



Note: If both the numerator and denominator of any fraction are multiplied by the same number the value of the fraction is unchanged.

$$\frac{1 \times 2}{2 \times 2} = \frac{2}{4} = \frac{1 \times 3}{2 \times 3} = \frac{3}{6} = \frac{1 \times 4}{2 \times 4} = \frac{4}{8}$$

Like and unlike fractions

Fractions having the same denominators are called **like fractions**. For example, 2/15, 8/15, 17/15, etc. are like fractions. Fractions with different denominators are called **unlike fractions**. For example, 2/13, 7/24, 9/125, etc, are unlike fractions.

Reducing Fractions to Lowest Terms

A fraction is in its simplest form (this is also called being expressed in lowest terms) if the Highest Common Factor (HCF) of the numerator and denominator is 1. For example, 1/2 is in lowest terms but 2/4 is not. To reduce a fraction to lowest terms (also called its simplest form), divide both the numerator and denominator by the HCF. For example, 2/3 is in lowest form, but 4/6 is not in lowest form (the HCF of 4 and 6 is 2) and 4/6 can be expressed as 2/3.

OPERATIONS WITH FRACTIONS

Adding fractions

One can easily add fractions if the denominators are the same. Simply add the numerators.

Example

$$\frac{1}{16} + \frac{3}{16} + \frac{5}{16} = \frac{9}{16}$$

If the fractions have different denominators, determine the **lowest common denominator** (LCD) so that you can express each fraction in equivalent form. The lowest common denominator is the lowest number that the denominator of each individual fraction will divide into.

Example

$$\frac{3}{5}+\frac{1}{9}+\frac{7}{15}=\frac{27}{45}+\frac{5}{45}+\frac{21}{45}=\frac{53}{45}=1\frac{8}{45}$$

Mixed numbers need to be converted to improper fractions before using this addition technique.

Subtracting fractions

Subtracting fractions is very similar to adding in that all fractions in your calculation need to have the same denominator. You may need to determine the lowest common denominator (LCD) as you did with addition.

Example: Find the value of $\frac{13}{16}+\frac{7}{12}-\frac{5}{6}-\frac{11}{32}$

$$\frac{78+56-80-21}{96}=\frac{134-101}{96}=\frac{33}{96}=\frac{11}{33}$$

Multiplying fractions

Multiplication of fractions is carried out by multiplying all the numerators together and all the denominators together.

$$\text{Product of two fractions} = \frac{\text{Product of their numerators}}{\text{Product of their denominators}}$$

$$\frac{2}{5}\times\frac{3}{7}=\frac{2\times 3}{5\times 7}=\frac{6}{35}$$

Sometimes the answer to a multiplication calculation can be simplified by dividing both numerator and denominator by the **same number**. This process is called **cancelling**, because you strike out the original numbers to create the new fraction. Example

$$\frac{\cancel{15}}{\cancel{25}}=\frac{3}{5}\ \begin{array}{l}\text{(divide numerator by 5)}\\ \text{(divide denominator by 5)}\end{array}$$

Sometimes for indicating multiplication of two fractions, we use the word **'of'**.

Examples

Question	Answers
(i) 3/5 of a dollar	3/5 x 100 = 60 cents.
(ii) 3/4 of a year	$\frac{3}{4}\times 12$ of a year = 9 months
(iii) 2/3 of a day	$\frac{2}{3}\times 24$ of a day = 16 hours

Dividing Fractions

To divide by a fraction, **invert it and multiply.**

Example: Divide $\frac{1}{2}$ by $\frac{1}{4}$

$$\frac{1}{2} \div \frac{1}{4} = \frac{1}{2} \times \frac{4}{1} = 2$$

Divide $3\frac{3}{4}$ by $\frac{5}{8}$

Convert mixed numbers to improper fractions.

$$3\frac{3}{4} \div \frac{5}{8} = \frac{15}{4} \div \frac{5}{8} = \frac{15}{4} \times \frac{8}{5} = \frac{120}{20} = \frac{6}{1} = 6$$

Comparing Fractions

Example: Which fraction is greater? $\frac{1}{4}$ **or** $\frac{7}{27}$**?**

1. Multiply the numerator of the first with the denominator of the second: $1 \times 27 = 27$
2. Now, multiply the denominator of the first with numerator of the second: $4 \times 7 = 28$ Since $28 > 27$, the second fraction 7/27 is greater than the first 1/4.

Example: Which is greater: $\frac{3}{7}$ **or** $\frac{7}{17}$**?**

Solution:

$$3 \times 17 \text{ and } 7 \times 7$$

$$51 > 49$$

Hence, $\frac{3}{7}$ is greater than $\frac{7}{17}$.

WORD PROBLEMS ON FRACTIONS

Q1. Saeed solved 2/7 part of an exercise while Aslam solved 4/5 of it. Who solved less?

In order to know who solved less part of the exercise, we will compare 2/7 and 4/5

LCM of denominators (i.e. 7 and 5) = 7 × 5 = 35

Converting each fraction in to an equivalent fraction having 35 as its denominator, we have 2/7 = (2 × 5)/(7 × 5) = 10/35 and 4/5 = (4 × 7)/(5 × 7) = 28/35

Since 10 < 28

Therefore, 10/35 < 28/35 => 2/7 < 4/5

Hence, Saeed solved lesser part than Aslam.

Note: This method can also be used to compare fractions in addition to the method discussed earlier.

Q2. Akram finished colouring a picture in 7/12 hour. Waqar finished colouring the same picture in 3/4hour. Who worked longer? By what fraction was it longer?

In order to know who worked longer, we will compare fractions 7/12 and 3/4.

LCM of 12 and 4 = 12

Converting each fraction into an equivalent fraction with 12 as denominator

7/12 = (7 × 1)/(12 × 1) = 7/12 and 3/4 = (3 × 3)/(4 × 3) = 9/12
Since 7 < 9
Therefore, 7/12 < 9/12 => 7/12 < 3/4
Thus, Waqar finished colouring in longer time.
Now, 3/4 - 7/12
= 9/12 – 7/12
= (9 – 7)/ 12
= 2/12 = 1/6
Hence, Waqar finished colouring in 1/6 hour more time than Akram.

Q3. Sami wants to put a picture in a frame. The picture is $7\frac{3}{5}$ cm wide. To fit in the frame the picture cannot be more than $7\frac{3}{10}$ cm wide. How much the picture should be trimmed?

Actual width of the picture = $7\frac{3}{5}$ cm = 38/5cm

Required width of the picture = $7\frac{3}{10}$ cm = 73/10 cm

Therefore, extra width = (38/5 – 73/10) cm
= (38 × 2)/(5 × 2) – (73 × 1)/(10 × 1) cm
= 76/10 – 73/10 cm
= (76 – 73)/10 cm
= 3/10 cm
Hence, 3/10 cm width of the picture should be trimmed.

Q4. A car runs 16 km using 1 litre of petrol car. How much distance will it cover using $2\frac{3}{4}$ litres of petrol?

In 1 litre, car runs 16 km

Therefore, in $2\frac{3}{4}$ litres of petrol car will travel = $2\frac{3}{4}$ × 16 km

= 11/4 × 16/1 km

$$= \frac{11}{{}_1\cancel{4}} \times \frac{\cancel{16}^{4}}{1}$$

= (11 × 4) km = 44 km

Hence, car travels 44 km in $2\frac{3}{4}$ litres of petrol.

5. Raza has read 3/4 of a book consisting of 288 pages. How many pages are still left?

Total number of pages in the book = 288

Number of pages read by Raza = 3/4 of 288 => 3/4 × 288

= 3/4 × 288/1

$$= \frac{3}{{}_1\cancel{4}} \times \frac{\cancel{288}^{72}}{1} = 3 \times 72 = 216$$

Therefore, number of pages left = 288 - 216 = 72

DECIMALS

A decimal number is a number with one or more digits to the right of the decimal point. The word "Decimal" really means "based on 10" (From Latin Decima: a tenth part). A "Decimal Number" usually means there is a Decimal Point. he place value of each digit corresponds to a power of 10. For example, the digits of the **number 7,532.418** have the following place values.

For the digits before the decimal point:

7 is in the thousands place

5 is in the hundreds place

3 is in the tens place

2 is in the ones, or units place

And, for the digits after the decimal point:

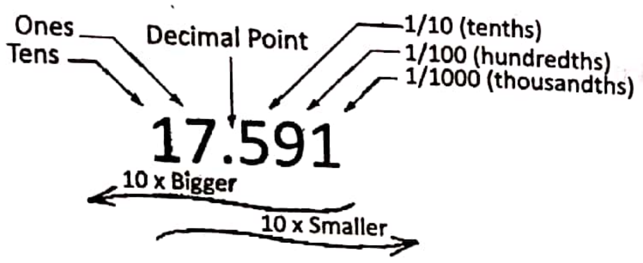
4 is in the tenths place

1 is in the hundredths place

8 is in the thousandths place

As we move right, each position is 10 times smaller. From Hundreds, to Tens, to Ones. What is **10 times smaller** than Ones? It is equal to 1/10ths (Tenths).

We can continue with smaller and smaller values, from **tenths**, to **hundredths**, and so on, like in this example:



Decimal System lets us write numbers as large or as small as we want, using the decimal point. Digits can be placed to the left or right of a decimal point, to show values greater than one or less than one. The **decimal point** is the most important part of a Decimal Number.

DECIMAL OPERATIONS

Additions

1. Align the number so that the tenths are written under tenths, the hundredths are written under hundredths.
2. The decimal point goes right under the decimal point in the numbers being added.

Example. **10.9 + 15.73+22.001**

Solution:

$$\begin{array}{r} 10.9 \\ 15.73 \\ +\underline{22.001} \\ 48.631 \end{array}$$

Subtraction

1. If one number has more decimal places than the other, add zeros to the right so that there are the same number of decimal places in the two numbers.
2. Align the numbers and put a decimal point under the decimal point in the number being subtracted.

Example. **17.19 – 8.4 = ?**

Solution:

$$\begin{array}{r} 17.19 \\ \underline{-8.40} \\ 8.79 \end{array}$$

Multiplication

1. Multiply the numbers as if they were whole numbers.
2. The number of decimal places in the answer is the sum of the number of decimal places in the factors.

Example. **3.1 × 2.7 = ?**

Solution: There are total two decimal places in the factors.
So, 31 × 27 = 837
Putting the decimal after two places, the answer is 8.37

Example. **75 × 3.3 = ?**

Solution: There will be one decimal places in the answer.
75 × 33 = 2475. So, the answer, after putting the decimal, will be 247.5
The last zeros in the decimals may be removed.

Division

1. Multiply the divisor by the power of 10 to make it a whole number.

2. Multiply the number being divided by the same power of 10.
3. The decimal point in the quotient is right above the decimal point in the number being divided.
4. Add zeros at the end of the number being divided to express the remainder as a decimal.

Example. $\frac{15}{1.2} = ?$

Solution: To make 1.2 a whole number, multiply by 10. Multiply 15 by 10 also and divide. Add one zero after the decimal point.

$$\frac{15}{1.2} = \frac{150}{12}$$

$$
\begin{array}{r}
12\,\overline{)\,150.0\,}\,(12.5 \\
\underline{12} \\
30 \\
\underline{24} \\
60 \\
\underline{60} \\
0
\end{array}
$$

Answer 12.5

Note: Adding zeros to the right of a decimal fraction does not change the value of the decimal fraction.

Decimal Fractions

Fractions in which denominators are powers of 10, are known as decimal fractions.

$\frac{1}{10} = 1$ tenth, $\frac{1}{100} = 1$ hundredth, $\frac{1}{1000} = 1$ thousandth etc.

$\frac{1}{100} = 1$ hundredth $= .01$, $\frac{2}{100} = 2$ hundredths $= .02$.

Rule for converting a Decimal into Vulgar Fraction

Put 1 in the denominator under the decimal point and annex with it as many zeros as is the number of digits after the decimal point in the numerator. Now, remove the decimal point and reduce the fraction to lowest terms.

Example: Convert the following into vulgar fractions

(i) 0.75 (ii) 3.004 (iii) 0056

Solution:

(i) $0.75 = \frac{75}{100} = \frac{3}{4}$ (ii) $3.004 = \frac{3004}{1000} = \frac{751}{250}$ (iii) $\frac{.0056}{10000} = \frac{7}{1250}$

Note 1: Annexing zeros to the extreme right of a decimal fraction does not change its value.
Thus, 0.8 = 0.80 = 0.800 etc.

Note 2: If numerator and denominator of a fraction contain the same number of decimal places, then we remove the decimal sign.

Example 2. **Simplify: (i)** $\frac{1.84}{2.99}$ **(ii)** $\frac{.365}{.584}$

Solution:

(i) $\frac{1.84}{2.99}=\frac{184}{299}=\frac{8}{13}$ (ii) $\frac{.365}{.584}=\frac{365}{584}=\frac{5}{8}$

EXERCISE

Q.1. **The total cost of $3\frac{1}{2}$kg of meat at Rs. 1.69 per kg and 20 lemons at Rs. 0.60 a dozen will be?**

Solution Find the cost of $3\frac{1}{2}$kg of meat:

$$\begin{array}{r} \text{Rs. } 1.69 \\ \times\ 3.5 \\ \hline 845 \\ 507\times \\ \hline \text{Rs. } 5.915 \end{array} = \text{Rs. } 5.92 \text{ approx}$$

Find the cost of 20 lemons:

Rs. 0.60 ÷ 12 = Rs. 0.05 (for 1 lemon)

Rs. 0.05 × 20 = Rs. 1.00 (for 20 lemons).

Add the cost of meat and the cost of lemons:

$$\begin{array}{r} \text{Rs. } 5.92 \\ +1.00 \\ \hline \text{Rs. } 6.92 \\ \hline \end{array}$$

Q.2. **A man willed his property to his three sons — to the youngest he gave Rs. 968.49, to the second 3.4 as much as to the youngest, and to the eldest 3.7 times as much as to the second. What was the value of his estate?**

Solution:

$$\begin{array}{rl} \text{Rs. } 968.49 & \text{willed to youngest son} \\ \times\ 3.4 & \\ \hline 387396 & \\ 2905470 & \\ \hline \text{Rs. } 3292.866 & \text{willed to the second son} \\ \times\ 3.7 & \\ \hline 23050062 & \\ 98795980 & \\ \hline \text{Rs. } 12183.6042 & \text{willed to the eldest son} \end{array}$$

Total Value of the estate

968.4900	Youngest son
3292.8700	Second son
+ 12183.6042	Eldest son
16444.9642	

Q.3. Which fraction is the largest: 1/2, 4/5, 5/8 and 7/10?

Solution: Write all the fractions with the same denominator.

LCM = 80

Multiply all the fractions with such a number that all has 80 as denominator.

(i) $\frac{7}{10}\times\frac{8}{8}=\frac{56}{80}$ **(ii)** $\frac{5}{8}\times\frac{10}{10}=\frac{50}{80}$

(iii) $\frac{4}{5}\times\frac{16}{16}=\frac{64}{80}$ **(iv)** $\frac{1}{2}\times\frac{40}{40}=\frac{80}{80}$

Now comparing all the fractions $\frac{64}{80}$ is the largest one.

∴ Hence 4/5 is the largest fraction.

Q.4. A car is run until the gas tank is 1/8 full. The tank is then filled to capacity by putting in 14 gal. The capacity of the gas tank of the car is?

Solution:

$$\frac{7}{8} \text{ of capacity} = 14 \text{ gal}$$

$$\therefore \text{ Full capacity} = 14\div\frac{7}{8}$$

$$= 14\times\frac{8}{7} = 16 \text{ gal}$$

Q.5. If 1/3 of the liquid contents of a can evaporates on the first day and 1/4 of the remainder evaporates on the second day, the fractional part of the original contents remaining at the close of the second day is

Solution:

First day: 1/3 evaporates, 2/3 remains

Second day: 1/4 of 2/3 evaporates

$\frac{3}{4}$ of $\frac{2}{3}$ remains

The amount remaining is

$$\frac{3}{4}\times\frac{2}{3}=\frac{6}{12}=\frac{1}{2} \text{ of original contents}$$

CHAPTER 21

ORDER OF OPERATIONS – PEMDAS

"Operations" means things like add, subtract, multiply, divide, squaring, etc. If it isn't a number it is probably an operation. But, when you see something like...

$$3 + 6 \times (5 + 4) \div 3 - 7$$

What part should you calculate first? Start at the left and go to the right? Or go from right to left?

Warning: Calculate them in the wrong order, and you will get a wrong answer!

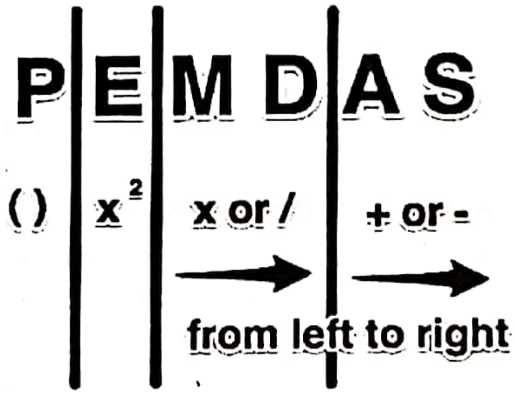
So, long ago people agreed to follow rules when doing calculations, and they are:

P	Parentheses first
E	Exponents (i.e. Powers and Square Roots, etc.)
MD	Multiplication and Division (left-to-right)
AS	Addition and Subtraction (left-to-right)

Divide and multiply rank equally (and go left to right).

Add and subtract rank equally (and go left to right).

After you have done "P" and "E", just go from left to right doing any "M" or "D" as you find them. Then, go from left to right doing any "A" or "S" as you find them. You can remember PEMDAS by saying "Please Excuse My Dear Aunt Sally".



Example 1. **Evaluate each expression using the rules for order of operations.**

	Expression	Evaluation	Operation
1.	6 + 7 x 8	= 6 + 7 x 8	Multiplication
		= 6 + 56	Addition
		= 62	
2.	16 ÷ 8 – 2	= 16 ÷ 8 – 2	Division
		= 2 – 2	Subtraction
		= 0	
3.	(25 - 11) x 3	= (25 - 11) x 3	Parentheses
		= 14 x 3	Multiplication
		= 42	

In Example 1, each problem involved only 2 operations.

Example 2. **Evaluate 3 + 6 x (5 + 4) ÷ 3 - 7 using the order of operations.**

Solution:

Step 1: 3 + 6 x (5 + 4) ÷ 3 – 7 = 3 + 6 x 9 ÷ 3 - 7 Parentheses

Step 2: 3 + 6 x 9 ÷ 3 – 7 = 3 + 54 ÷ 3 – 7 Multiplication

Step 3: 3 + 54 ÷ 3 – 7 = 3 + 18 – 7 Division

Step 4: 3 + 18 – 7 = 21 – 7 Addition

Step 5: 21 – 7 = 14 Subtraction

Example 3. **Evaluate 9 - 5 ÷ (8 - 3) x 2 + 6 using the order of operations.**

Solution:

Step 1: 9 - 5 ÷ (8 - 3) x 2 + 6 = 9 - 5 ÷ 5 x 2 + 6 Parentheses

Step 2: 9 - 5 ÷ 5 x 2 + 6 = 9 - 1 x 2 + 6 Division

Step 3: 9 - 1 x 2 + 6 = 9 - 2 + 6 Multiplication

Step 4: 9 - 2 + 6 = 7 + 6 Subtraction

Step 5: 7 + 6 = 13 Addition

Note: Speakers of British English use "**BODMAS**", instead of PEMDAS which stands for "Brackets, Orders, 'Division and Multiplication', and 'Addition and Subtraction'". Since "brackets" are the same as parentheses and "orders" are the same as exponents, the two acronyms mean the same thing.

Example 4. **A boy was asked to multiply a certain number by 53. He multiplied it by 35 and got his answer less than the correct one by 1206. Find the number to be multiplied.**

Solution: Let the required number be x. Then,

$53x - 35x = 1206$ or

$18x = 1206$ or

$x = 67$

Required number = 67

Example 5. **Simplify:** $-3x\left[\frac{1}{2}(3x - 2y) - 2\left(x(3+y)+4y\right)\right]$

Solution:

$$= -3x\left[\frac{1}{2}(3x - 2y) - 2(3x + xy + 4y)\right]$$

$$= -3x\left[\frac{3}{2}x - y - 6x - 2xy - 8y\right]$$

$$= -3x\left[-\frac{9}{2}x - 2xy - 9y\right]$$

$$= \frac{27}{2}x^2 + 6x^2y + 27xy$$

MULTIPLE CHOICE QUESTIONS

Find out the correct answer.

1. $5005 - 5000 \div 10.00 = ?$

(a) 0.5 (b) 50 (c) 5000 (d) 4505

2. Four children A, B, C and D divide a bag of sweets. A takes $\frac{1}{3}$ of them, B takes $\frac{2}{5}$th of the remainder and the rest is equally shared between C and D. What fraction of the sweets did C or D get?

(a) $\frac{1}{5}$ (b) $\frac{1}{17}$ (c) $\frac{1}{6}$ (d) $\frac{1}{4}$

3. A boy was asked to multiply a certain number by 25. He multiplied it by 52 and got his answer more than the correct one by 324. The number to be multiplied was:

(a) 12 (b) 15 (c) 25 (d) 32

4. What fraction must be subtracted from the sum of $\frac{1}{4}$ and $\frac{1}{6}$ to have an average of $\frac{1}{12}$ of all the three fractions?

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{1}{4}$ (d) $\frac{1}{6}$

5. **The smallest fraction which should be subtracted from the sum of $1\frac{3}{4}, 2\frac{1}{2}, 5\frac{7}{12}, 3\frac{1}{3}$ and $2\frac{1}{4}$ to make the result a whole number is**

(a) $\frac{5}{12}$ **(b)** $\frac{7}{12}$ **(c)** $\frac{1}{2}$ **(d)** 7

ANSWERS

1. **Ans.** **(d)**

Solution: Given Expression $= 5005 - 5000 \div 10$

$= 5005 - 500 = 4505$

2. **Ans.** **(a)**

Solution: A's share $= \frac{1}{3}$. Remainder $= \left(1-\frac{1}{3}\right) = \frac{2}{3}$

B's share $= \frac{2}{5}$ of $\frac{2}{3} = \frac{4}{15}$.

Rest $= \left(\frac{2}{3}-\frac{4}{15}\right) = \frac{6}{15} = \frac{2}{5}$

C's share = D's share $= \frac{1}{2}$ of $\frac{2}{5} = \frac{1}{5}$

3. **Ans.** **(a)**

Solution: Let the required number be x. Then,

$52x - 25x = 324$ or $27x = 324$ or $x = 12$

$\therefore$ Required number = 12

4. **Ans.** **(d)**

Solution:

$$\frac{1}{4}+\frac{1}{6}-x=3\times\frac{1}{12}$$

$$\frac{1}{4}+\frac{1}{6}=\frac{3}{12}+x$$

$$\frac{6+4}{24}=\frac{3}{12}+x$$

$$\frac{5}{12}-\frac{3}{12}=x$$

$$\frac{2}{12}=x$$

$$x=\frac{1}{6}$$

5. **Ans.** (a)

Solution: Sum of given numbers $=\frac{7}{4}+\frac{5}{2}+\frac{67}{12}+\frac{10}{3}+\frac{9}{4}$

$$=\left(\frac{21+30+67+40+27}{12}\right)=\frac{185}{12}$$

The whole number just less than $\frac{185}{12}$ is 15.

Let $\frac{185}{12}$ - $x=15$. Then, $x=\left(\frac{185}{12}-15\right)=\frac{5}{12}$

5/12 must be subtracted from 185/12 to make it a whole number

SELF-EXERCISE

Use order of operations to simplify the following expressions:

1. $21+(4+10)$
2. $(4-1)^2+(12-3)^2+(5-2)^3$
3. $16+(4\times 10)$
4. $6(5-3)^2$
5. $20(3+7)$
6. $5[5-(0\times 3)]$
7. $\frac{36-6}{12+3}$
8. $4\times 3+4\times 5\div 4-1$
9. $\frac{11+9}{2\times 5}$

10. $[7+(4\cdot 2)]\times 2$

11. $[16-(2\cdot 3)]\times 3$

12. $[(7\cdot 4)-(3\cdot 6)]\div 2$

13. $[6+(4\cdot 2)]+[(7\cdot 3)-4]$

14. $\{[(3\cdot 2)+4]+5\}\cdot 2$

15. $\{[(2\cdot 3)+(7\cdot 4)]+2\}\cdot[(8-6)-(4-3)]$

16. $3\{5+[(2+7)+11]\}$

17. $[(16-4)\div(3\times 2)]\div 3$

18. $6+[12-(4\times 2)]$

19. $[8+(2\times 1)]\times 0$

20. $[40\div(4\times 5)]\div 2$

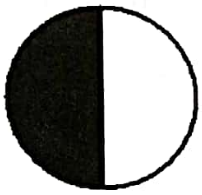
CHAPTER 22

PERCENTAGE

Percentages or **percent** refer to fractions of a whole; "that is", the percentage is how much of the whole thing you have. For instance, "50%" means "½", "25%" means "¼", etc. Percent is actually per cent, meaning out of a hundred. Because "percent" means "per 100". "This should be divided by 100". So, 75% really means 75/100

- And 100% is 100/100, or exactly 1 (100% of any number is just the number unchanged)
- And 200% is 200/100, or exactly 2 (200% of any number is twice the number)

A percent can also be expressed as a *Decimal* or a *Fraction*



1/2

A Half can be written...

As a percentage: **50%**

As a decimal: **0.5**

As a fraction: $\mathbf{\frac{1}{2}}$

Some other common percentage amounts, and their fraction and decimal equivalents.

Percent	Decimal	Fraction
1%	0.01	1/100
5%	0.05	1/20
10%	0.1	1/10
12½%	0.125	1/8
20%	0.2	1/5
25%	0.25	1/4
331/3%	0.333...	1/3
50%	0.5	1/2
75%	0.75	3/4
80%	0.8	4/5
90%	0.9	9/10
100%	1	1 or 1/1
125%	1.25	5/4

150%	1.5	3/2
200%	2	2 or 2/1

Decimal-Percent-Fraction Conversions

Decimal to Percent: Decimal to Percent conversions are easy; just move the decimal point two places to the right and insert the percent symbol. For example,

$$0.25 = 25\%$$

$$2.55 = 255\%$$

$$0.0075 = 0.75\%$$

Note: That 0.75% is less than one percent. It should not be confused with 75%, which is 0.75 as a decimal.

Percent to Decimal: Percent to Decimal conversions are easy; just move the decimal point two places to the left and drop the % symbol. For example:

$$34\% = 0.34$$

$$106\% = 1.06$$

$$0.5\% = 0.005$$

Percent to Fraction: Percent to Fraction conversions aren't too bad. Convent the percent to a decimal first, and then to a fraction. For example:

$$40\% = 0.40 = \frac{40}{100}$$

Now simply the fraction: 40/100 = 4/10 = 2/5

Most conversions are easy like the above, but some require a little extra care. For example,

$$105\% = 1.05 = \frac{105}{100} = \frac{21}{20}$$

Convert a fraction into a per cent: To convert any fraction $\frac{1}{m}$ to rate per cent, multiply it by 100 and put % sign, i.e. $\frac{1}{m} \times 100\%$

What percentage is equivalent to $\frac{3}{5}$

Solution: $\frac{3}{5} \times 100\%$ **= 60 %**

Percent Problems

Rule 1: Percent · Base = Amount

Percent problems can be solved by writing equations. An equation uses an equal sign (=) to show that two mathematical expressions have the same value.

Percent **of** the Base **is** the Amount.

Percent · Base = Amount

In the examples below, the unknown is represented by the letter x.

Example: **Write an equation that represents the following problem:**
30 is 20% of what number?

20% of what number is 30?

Rewrite the problem in the form "percent of base is amount."

Percent is: 20%
Base is: unknown
Amount is: 30

Identify the percent, the base, and the amount.

Percent · Base = Amount

$20\% \cdot x = 30$

Write the percent equation. Using x for the base, which is the unknown value.

When you have an equation such as 20% of $x = 30$, you can divide 30 by 20% to find the unknown.

$$x = 30 \div 20\%$$

$$x = 30 \div 20\% = 30 \div 0.20 = 150$$

Example: **(i) What is 35% of 160?**
(ii) 54 is 45% of what number?
(iii) 77 is what per cent of 140?

In each case, there is one unknown. Translate each sentence, replacing "is" by "=" and the unknown by x.

Solution:

(i) $x = 35\%$ of $160 \Rightarrow x = (0.35)(160) = 56$

(ii) $54 = 45\%$ of $x \Rightarrow 54 = 0.45x \Rightarrow x = 54 \div 0.45 = 120$

(iii) $77 = x\%$ of $140 \Rightarrow 77 = \frac{x}{100}(140) \Rightarrow 77(100) = 140x \Rightarrow x = \frac{7700}{140} = 55\%$

Example: 24 students in a class took an algebra test. If 18 students passed the test, what percent did not pass?

Solution: Set up the problem like this:

First, find out how many student did not pass.

Number of students who did not pass is 24 - 18 = 6

Then, write down the following equation:

$x\%$ of $24 = 6$ or $x\%$ times $24 = 6$

To get $x\%$, just divide 6 by 24

$6/24 = 0.25 = 25/100 = 25\%$

Therefore, 25% of students did not pass the algebra test.

Rule 2: Percent Increase and Decrease

Subtract the old from the new, then divide by the old value. Show that as a Percentage.

$$= \frac{\textit{new value} - \textit{old value}}{\textit{Old Value}} \times 100\%$$

The per cent increase of a quantity is $\frac{\text{the actual increase}}{\text{the original amount}} \times 100\%$

The per cent decrease of a quantity is $\frac{\text{the actual decrease}}{\text{the original amount}} \times 100\%$

Example: A pair of socks went from \$5 to \$6, what is the percentage change?

Solution: (Method 1)

Step 1: \$5 to \$6 is a \$1 increase

Step 2: Divide by the old value: \$1/\$5 = 0.2

Step 3: Convert 0.2 to percentage: 0.2×100 = 20% rise.

Answer (Method 2)

Step 1: Divide new value by old value: \$6/\$5 = 1.2

Step 2: Convert to percentage: 1.2×100 = 120% (i.e. \$6 is 120% of \$5)

Step 3: Subtract 100%: 120% - 100% = 20%, and that means a 20% rise.

Example: There were 160 candies in the box yesterday, but now there are 116, what is the percentage change?

Solution: 160 to 116 is a decrease of 44. Compared to yesterday's value: 44/160 = 0.275 = 27.5% decrease.

Price Discount

Price discount occurs when items are sold at a reduced cost. Find out now how you can compute the discount.

Terms to know:

List price: Regular price of an item.

Discount: A reduction made from the regular price.

Sale price: The reduced cost of an item.

Discount rate: Percent that the price is reduced.

To find a sale price, use the following formulas:

Sale price = List Price - Discount

Discount = (List Price) × (Discount Rate)

Example: Find the sale price for an item that has a list price of 100 dollars and a discount rate of 25%.

Solution: Discount = 100 × 25% = 100 × 0.25 = 25

Sale price = List Price - Discount Price = 100 - 25 = 75 dollars.

How to calculate sales tax

To calculate sales tax of an item, simply multiply the cost of the item by the tax rate.

Example: You buy a car for 20,000 dollars and pay 5% in tax. How much is tax?

Solution: Tax = 20,000 times 0.05 = 1,000 dollars.

The cost with tax is 20,000 + 1000 = 21,000.

Example: The list price of a scientific calculator is 100 dollars. If you get a price discount of 15% and pay 5% for tax, what is the actual cost of your calculator?

Solution: First, find the price after the discount

Discount = 100 times 15% = 100 times 0.15 = 15

Sale price = 100 - 15 = 85 dollars

Tax paid = 85 times 5% = 85 times 0.05 = 4.25

Actual cost = 85 + 4.25 = 89.25 dollars.

Reverse percentages

Sometimes a question can be asked to work backwards and find the original price of something after the price has increased. Some people think that a percentage increase can be "reversed" by the same percentage decrease. But no!

Example: 10% of 100

A 10% increase from 100 is an increase of 10, which equals 110 ...

... but a 10% reduction from 110 is a reduction of 11 (10% of 110 is 11)

So we ended up at 99 (not the 100 we started with)

What happened?

The 10% increase was applied to 100

But the 10% decrease was applied to 110

Example: A radio sells for £63, after a 40% increase in the cost price. Find the cost price.

Solution: Start with the original amount as 100%.

Cost price = 100%

We are told the selling price is a 40% in the cost price.

So the selling price is 100% + 40% = 140% of the cost price.

We know that the selling price is £63, so 140% = £63.

Now calculate 1%.

140% = £63

1% = £63/140

1% = £0.45

The cost price is 100%, so multiply £0.45 by 100.

Cost price = 0.45 × 100 = £45.

Example: A new car falls in value by 30% in a year. After a year, it is worth £8,400.

Find the price of the car when it was new.

Solution: Remember that the original price of the car is 100%.

Original price = 100%.

Second-hand price = 100% - 30% = 70%.

So £8,400 = 70% of the original price.

So 1% of original price = £8,400 ÷ 70

Original price = 100% = 100 x 1% = 100 x (£8,400 ÷ 70) = £12,000.

It is easy to go wrong in this type of question. Always check that your answer is realistic.

Cumulative Increase Decrease and Compound Interest

Example: £500 is put in a bank where there is 6% per annum interest. Work out the amount in the bank after 1 year.

Simple interest formula = A= P (1+r t)

Where A is the Final investment value

P is the Principal or original amount

r is the interest rat% and

t is the time period There is a simple interest formula which allows as to calculator interest earned or charged

$$\begin{aligned} A &= P\,(1+rt) \\ A &= 500\,(1+6\% \times 1) \\ &= 500\,(1+0.06) \\ &= 500\,(1.06) \\ &= 530 \end{aligned}$$

If in this example, the money was left in the bank for another year, the £530 would increase by 6%. The profit, therefore, will be higher than the previous year (6% of £530 is more than 6% of £500). Every year, if the money is left sitting in the bank account, the amount of interest paid would increase each year. This phenomenon is known as **compound interest.**

The simple way to work out compound interest is:

$$\text{Compound Interest} = (1 + \text{interest rate }\%)^{\text{no of years}} \times \text{original value}$$

So if the £500 had been left in the bank for 9 years, the amount would have increased to:

$$500 \times (1.06)^9 = £845$$

There can be a question in final exam, in which examiner may ask cumulative increase or decrease in a quantity.

- To increase a number by r%, multiply it by $(1 + r\%)^{\text{no of years}}$.
- To decrease a number by r%, multiply it by $(1 - r\%)^{\text{no of years}}$.

Example: At the end of 2016 there were 5000 members of a certain rare breed of animal remaining in the world. It is predicted that their number will decrease by 12% each year. How many will be left at the end of 2020?

At the end of 2017, there will be (100 - 12)/100 × 5000 = 4400

The compound interest formula above can also be used for percentage decreases. So after 4 years, the number of animals left would be:

$$5000 \times (1-0.12)^4 = 2998$$

Example: From January 1, 1990 to January 1, 2000, the population of a town increased by 5% every year. If the population was 5000 on January 1, 1990, what was the population of January 1, 2000?

Solution: Since the population on January 1, 1990 was 5000

After exactly 10 years, the population on January 1, 2000 was = $(1+5\%)^{10} \times 5000$

$$(1.05)^{10}(5000) = (1.6289)(5000) = 8144$$

Example: The population of a town is 176,400. If it increases at the rate of 5% per annum, what will be its population 2 years hence?

Solution: Population after 2 years = $176400 \times \left[1+\dfrac{5}{100}\right]^2$

$$= \left[176400 \times \frac{21}{20} \times \frac{21}{20}\right] = 194481$$

Population 2 years ago $= \dfrac{176400}{\left[1+\dfrac{5}{100}\right]^2}$

$$= \left[176400 \times \frac{20}{21} \times \frac{20}{21}\right] = 160000$$

Advance Rules of Percentage

(a) If A is x% more than that of B, then B is less than that of A by

$$\left[\frac{x}{100+x} \times 100\right]\%$$

(b) If A is x% less than that of B, then B is more than that of A by

$$\left[\frac{x}{100-x} \times 100\right]\%$$

Example: If A's income is 40% less than B's income, then how much per cent is B's income more than A's income?

Solution: Here, x = 40

$\therefore$ Required answer = $\left(\dfrac{x}{100-x} \times 100\right)\%$

$$= \left(\frac{40}{100-40} \times 100\right)\% = 66\frac{2}{3}\%$$

Similarly

(a) If two numbers are respectively $x\%$ and $y\%$ more than a third number, then the first number is $\left(\dfrac{100+x}{100+y} \times 100\right)\%$ of the second and the second is $\left(\dfrac{100+y}{100+x} \times 100\right)\%$ of the first.

(b) If two numbers are respectively x% and y% less than a third number, then the first number is $\left(\dfrac{100-x}{100-y} \times 100\right)\%$ of the second and the second is $\left(\dfrac{100-y}{100-x} \times 100\right)\%$ of the first.

Example: Two numbers are respectively 20% and 50% more than a third number. What per cent is the first of the second?

Solution: Here, $x = 20$ and $y = 50$

$$\therefore \quad \text{First number} = \left(\frac{100+x}{100+y}\right)\times 100\% \text{ of the second}$$

$$= \left(\frac{100+20}{100+50}\right)\times 100\% \text{ of the second}$$

i.e. 80% of the second number.

EXERCISE

1. A camera costs £180 in a 10% sale. What was the pre-sale price?
2. After fuel prices rose by 15%, a family's annual electricity bill was £1654. What would the bill have been without the price increase?
3. The cost of a product, including VAT at 20% is £540. What is the pre-VAT price?
4. The sale price of a television is £420 after a 15% reduction. What was the price before the sale?
5. After a 6.5% pay rise an engineer's salary is £36,700. What was the salary before the increase?

ANSWERS

(1)	£200	(2)	£1,438.26	(3)	£450	(4)	£494.11	(5)	£34,460.09

MULTIPLE CHOICE QUESTIONS

Find out the correct answer.

1. **What percentage is equivalent to $5\frac{1}{4}$?**

   (a) 525% (b) 425% (c) 625% (d) None of these

2. **$6\frac{2}{3}$% expressed as a fraction in its lowest term is**

   (a) $\frac{2}{15}$ (b) $\frac{1}{15}$ (c) $\frac{3}{20}$ (d) None of these

3. **What per cent of 12 is 84?**

   (a) 800% (b) 600% (c) 700% (d) None of these

4. **If 200% of a number is 90, then what is the 80% of that number?**

   (a) 48 (b) 36 (c) 24 (d) None of these

5. **20% of 30% of 20% of Rs. 850 is**

   (a) Rs. 9.50 (b) Rs. 10.20 (c) Rs. 10.50 (d) None of these

6. **If 12% of x is equal to 6% of y, then 18% of x will be equal to how much per cent of y?**
(a) 7% (b) 9% (c) 11% (d) None of these

7. **A number is increased by 20% and then decreased by 20%, the final value of the number**
(a) Does not change (b) Decreases by 2%
(c) Increases by 4% (d) Decreases by 4%

8. **The population of a town is decreased by 20% and 25% in two successive years. What per cent population is decreased after two years?**
(a) 50% (b) 40% (c) 60% (d) None of these

9. **For a rectangle, the length and breadth are increased by 10% and 20%, respectively. The percentage increase in area is**
(a) 24% (b) 48% (c) 32% (d) None of these

10. **The income of a company increases 20% per annum. If its income is Rs. 2,664,000 in the year 2015 what was its income in the year 2013?**
(a) Rs. 17,50,000 (b) 16,50,000 (c) Rs. 18,50,000 (d) None of these

ANSWERS

1. **Ans.** **(a)**

Solution:

$$5\frac{1}{4}=\frac{21}{4}=\frac{21}{4}\times 100=525\%$$

2. **Ans.** **(b)**

Solution:

$$6\frac{2}{3}\%=\frac{\left(\frac{20}{3}\right)}{100}=\left(\frac{20}{3}\times\frac{1}{100}\right)=\frac{1}{15}$$

3. **Ans.** **(c)**

Solution:

Let x% of 12 = 84

$$\Rightarrow \quad \frac{x}{100}\times 12=84\Rightarrow x=\frac{84\times 100}{12}=700$$

$\therefore$ 700% of 12 is 84.

4. **Ans.** **(b)**

Solution:

$$\text{Let } 200\% \text{ of } x=90\Rightarrow \frac{200}{100}\times x=90$$

$$\Rightarrow \quad x=\frac{100\times 90}{200}=45$$

$$\therefore \quad 80\% \text{ of } 45 = \frac{80}{100} \times 45 = 36$$

5. **Ans.** **(b)**

Solution:

$$20\% \text{ of } 30\% \text{ of } 20\% \text{ of } 850 = \frac{20}{100} \times \frac{30}{100} \times \frac{20}{100} \times 850 = \frac{1020}{100} = 10.20$$

6. **Ans.** **(b)**

Solution:

We have, 12% of x = 6% of y

$\Rightarrow$ 2% of x = 1% of y

$\Rightarrow$ (2 × 9)% of x = (1 × 9)% of y

$\Rightarrow$ 18% of x = 9% of y

7. **Ans.** **(d)**

Solution:

Here, $x = 20$ and $y = -20$

$\therefore$ The net % change in value

$$= \left(x + y + \frac{xy}{100}\right)\%$$

$$= \left(20 - 20 - \frac{20 \times 20}{100}\right)\% \text{ or } -4\%$$

Since the sign is –ve, there is decrease in value by 4%

8. **Ans.** **(b)**

Solution:

Here, $x = -20$ and $y = -25$

$\therefore$ The net % change in population

$$= \left(x + y + \frac{xy}{100}\right)\%$$

$$= \left(-20 - 25 + \frac{20 \times 25}{100}\right) \text{or} - 40\%$$

Since the sign is –ve, the population is decreased by 40% after two years.

9. **Ans.** **(c)**

Solution:

Since side$_1$ × side$_2$ = area

$\therefore$ Net % change in area $= \left(x + y + \frac{xy}{100}\right)\%$

$$= \left(10 + 20 + \frac{10 \times 20}{100}\right)\% = 32\%$$

$\therefore$ The area of the rectangle increases by 32%.

10. **Ans.** **(c)**

Solution: Here, P = 2664000, r = 20 and n = 2

∴ Company's income in the year 2015

$$= \frac{P}{\left(1+\frac{r}{100}\right)^n} = \frac{2664000}{\left(1+\frac{20}{100}\right)^2}$$

$= 2664000/\ (1.2)^2 =$ Rs. 18,50,000

SELF-EXERCISE

Find the Percentage

1. 1% of 100 + 99% of 100=?
2. 10% of 50 + 80% of 5 =?
3. 12% of 100 x 11% of 100=?
4. 40% of 80 - 16% of 25=?
5. 40% of 45 - 45% of 40=?
6. 2% of 100 x 60% of 100=?
7. 25% of 44 + 15% of 80=?
8. A property dealer sold a house for Rs. 600,000 and charged 1% commission from both the parties. How much did the seller get? How many rupees did the buyer pay? How much commission did the agent get?
9. A property dealer sold a property worth Rs.15, 00,000 and charged 3% commission from both the parties. How much did the seller get? How many Rupees did the buyer pay? How much commission did the agent get?
10. A commission agent sold a house for Rs.5, 00,000 and charged a commission at 2% from both the seller and the buyer. What did the seller get? What did the buyer pay? What did the agent get?
11. A newspaper agent sold newspapers for Rs. 10800 in a month and got a total commission of Rs.1620. Find the percentage of his commission.
12. A factory owner gives 15% commission on the written price to a shopkeeper. What will the shopkeeper pay in cash for an article for which written price is Rs. 2000?
13. Ali is a salesman in a shop. His salary is Rs. 7000 per month plus a commission @ 3% on the price of sold goods. He sold goods worth Rs. 50,000 in a month. Find his total income for the month.
14. Aslam sold cotton of Abbas worth Rs. 1000,000. He deducted 5% commission. What did Aslam get?
15. A manufacturer gives 10% commission on the written price to a shopkeeper. What does the shopkeeper pay in cash for an article with written price as Rs.1500?
16. Mr. Rehman ordered a car worth Rs. 900,000 and was given a discount. Given that he paid Rs. 855,000 for his new car. What percentage discount he received?
17. Seemab buys 5 kg of tea at Rs. 125.00 per kg. In addition, for every kilogram of tea purchased, she has to pay a sales tax of 6% on the selling price. Calculate the total amount of money that she had to pay.

18. Murad bought his house for Rs. 300,000 and sold it for Rs. 600,000. What was the percentage of increase?

19. Nadeem spends 30% of monthly income on food articles, 40% of the remaining on conveyance and clothes and saves 50% of the remaining. If his monthly salary is Rs. 18400, how much money does he save every month?

20. Write these decimals as fractions:

0.3 = ... 0.5 = ... 0.6 = ...

0.05 = ... 0.25 = ... 0.36 = ...

21. Write these fractions as decimals:

$\frac{7}{10}$ = ... $\frac{1}{5}$ = ... $\frac{2}{5}$ = ...

$\frac{7}{8}$ = ... $\frac{2}{3}$ = ... $\frac{9}{20}$ =..................

22. Write these percentages as decimals:

3% = ... 30% = ... 25% = ...

8% = ... 12% = ... 67% = ...

23. Write these percentages as fractions:

20% = ... 75% = ... 5% = ...

40% = ... 15% = ... 24% = ...

24. Write these decimals as percentages:

0.25 = ... 0.5 = ... 0.7 = ...

0.45 = ... 0.09 = ... 0.4 = ...

25. Write these fractions as percentages:

$\frac{1}{10}$ = ... $\frac{1}{5}$ = ... $\frac{9}{10}$ = ...

$\frac{4}{5}$ = ... $\frac{17}{20}$ = ... $\frac{1}{3}$ = ...

CHAPTER 23

RATIO AND PROPORTION

RATIO

A "ratio" is just a comparison between two different things. For instance, someone can look at a group of people and refer to the "ratio of men to women" in the group. Suppose there are thirty-five people, fifteen of whom are men. Then the ratio of men to women is 15 to 20.

Notice that, in the expression "the ratio of men to women", "men" came first. This order is very important, and must be respected: whichever word came first, its number must come first. If the expression had been "the ratio of women to men", then the numbers would have been "20 to 15". When we express ratios in words, we use the word "to"--we say "the ratio of something to something else." Ratios can be written in several different ways: as a fraction, using the word "to", or with a colon.

Using the ":" to separate the values: 3 : 1
Instead of the ":" we can use the word "to": 3 to 1
Or write it like a fraction: 3/1

Example: In a certain class, the ratio of passing grades to failing grades is 7 to 5. How many of the 36 students failed the course?

The ratio, "7 to 5" (or 7:5 or 7/5), tells that, of every 7 + 5 = 12 students, five failed.

That is, 5/12 of the class flunked. Then (5/12)(36) = 15 students failed.

Example: In a park, the ratio of ducks to geese is 16 to 9. How many of the 300 birds are geese?

The ratio tells me that, of every 16 + 9 = 25 birds, 9 are geese.

That is, 9/25 of the birds are geese.

Then there are (9/25)(300) = 108 geese.

Let's look at a classroom: there are 16 boys and 15 girls.

Ratios can be part-to-part: $\frac{16\,boys}{15\,girls}$

Ratios can be part-to-whole: $\frac{16\ boys}{31\,students}$

If a ratio is part-to-whole, you can divide and find a decimal or a percent.

$$\frac{16\ boys}{31\ students}$$

16/31 = 0.516, or 51.6% are boys

Example: Express the ratio in simplest form: 240 miles to 8 gallons

When you simplify, you get (240 miles) / (8 gallons) = (30 miles) / (1 gallon), or, in more common language, 30 miles per gallon.

Example: Express the ratio in simplest form: $10 to $45

This exercise wants me to write the ratio as a reduced fraction: .10/45 = 2/9.

SELF-EXERCISE

Simplify the following ratios:-

1. 100: 200
2. 100: 300
3. 100: 500
4. 200: 1000
5. 300: 600
6. 50: 100
7. 45: 180
8. 15: 150
9. 12: 12
10. The ratio of 3.5 kg to 280 gram is?
11. The ratio of 1.75 kg to 150 gram is?
12. The ratio of 500 g to 250 gram is?
13. The ratio of 900 g to 225 gram is?
14. The ratio of 1 kg to 100 gram is?
15. 14. The ratio of 1.5 km to 500 m is?
16. Ali gets one-fourth of his pay as house rent allowance. If he gets Rs. 48,000 as pay, what is his allowance?
17. Yasir gets one-eighth of his pay as medical allowance. If he gets Rs. 34,000 as pay, what is his allowance?
18. Sadaf gets one-fifth of his pay as house rent allowance. If he gets Rs. 9,000 as allowance, what is his pay?
19. Rabia puts two-thirds of her income in her saving account. If she saves Rs. 43,000, what is her income?
20. An employ gets one-fourth of his pay as house rent allowance. If he gets Rs. 12,000 as allowance, what is his pay?

PROPORTION

The equality of two ratios is called proportion. A proportion is two ratios that have been set equal to each other; a proportion is an equation that can be solved. For instance, 5/10 equals 1/2. Solving a proportion means that you are missing one part of one of the fractions, and you need to solve for that missing value. For instance, suppose you were given the following equation:

$$\frac{x}{10}=\frac{1}{2}$$

You can solve the equation by multiplying through on both sides by 10 to clear the denominators:

$$\frac{x}{10}=\frac{1}{2}$$

$$10\left(\frac{x}{10}\right)=10\left(\frac{1}{2}\right)\Rightarrow x=5$$

A proportion is read as "x is to y as z is to w"

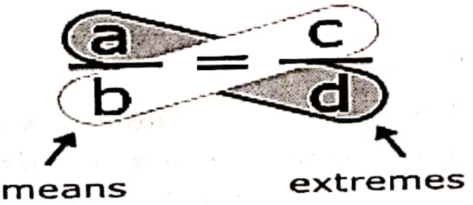
$$\frac{x}{y}=\frac{z}{w}$$

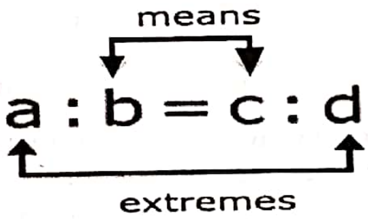
Where y, w ≠ 0

xw = yz

If a : b = c : d, we write, a : b :: c : d and we say that a, b, c, d are in proportion. In a proportion, the first and fourth terms are known as **extremes**, while second and third terms are known as **means**.

RULE: In a true proportion, the product of the means equals the product of the extremes.





Example: Find the unknown value in the proportion: $2:x=3:9$

Convert the colon-based odds-notation ratios to fractional form: $\frac{2}{x}=\frac{3}{9}$

Solve the proportion:

$$\frac{2}{x}=\frac{3}{9}$$

$$9(2)=x(3)$$

$$18=3x$$

$$6=x$$

Using Proportions to Solve Percents

A percent is actually a ratio! Saying "25%" is actually saying "25 per 100":

25% = 25/100

We can use proportions to solve questions involving percent.

First, put what we know into this form: Part/Whole = Percent/100

Example: What is 25% of 160?

The percent is 25, the whole is 160, and we want to find the "part":

Part/160 = 25/100

Multiply across the known corners, then divide by the third number:

Part = (160 × 25) / 100 = 4000 / 100 = 40

Answer: 25% of 160 is 40.

Example: Divide Rs. 1162 among A, B, C in the ratio 35 : 28 : 20.

Solution: Sum of the terms of the ratio = (35 + 28 + 20) = 83

$$\therefore \quad \text{A's share} = \text{Rs.}\left(1162\times\frac{35}{83}\right) = \text{Rs. } 490$$

$$\text{B's share} = \text{Rs.}\left(1162\times\frac{28}{83}\right) = \text{Rs. } 392$$

$$\text{C's share} = \text{Rs}\left(1162\ X\frac{20}{83}\right) = \text{Rs. } 280$$

Example: A mixture contains alcohol and water in the ratio 4 : 3. If 5 litres of water is added to the mixture, the ratio becomes 4 : 5. Find the quantity of alcohol in the given mixture.

Solution: Let the quantity of alcohol and water be 4x and 3x, respectively.

$$\text{Then, } \frac{4x}{3x+5}=\frac{4}{5} \Leftrightarrow 20x = 12x + 20 \Leftrightarrow x = 2.5$$

∴ Alcohol in given mixture = 4x litres = 4 × 2.5 = 10 litres

Example: If a machine makes 3 yards of cloth in 2 minutes, how many yards of cloth will the machine make in 50 minutes?

Solution: The amount of cloth is proportional to the time the machine operates. Let y be the number of yards of cloth the machine makes in 50 minutes; then $\frac{2\text{ minutes}}{50\text{ minutes}}=\frac{3\text{ yards}}{y\text{ yards}}$, so $\frac{2}{50}=\frac{3}{y}$. Cross multiply and you have 2y = 150, so y = 75.

Therefore, the machine makes 75 yards of cloth in 50 minutes.

THE THEOREM OF THE ALTERNATE PROPORTION

The numbers in a proportion are called the terms: the 1st, the 2nd, the 3rd, and the 4th.

1st : 2nd = 3rd : 4th

We say that the 1st and the 3rd are corresponding terms, as are the 2nd and the 4th.

The following is the theorem of the alternate proportion: *If four numbers are proportional, then the corresponding terms are also proportional.* That is, as the first term is to the third, so the second will be to the fourth.

If

$$a : b = m : n,$$

then, alternately,

$$a : m = b : n.$$

Since

$$1 : 3 = 5 : 15,$$

then alternately,

$$1 : 5 = 3 : 15.$$

Example: Complete this proportion, 5 : 8 = 35 : ?

Look at it alternately. 35 is seven times 5. Therefore the missing term will be seven times 8, which is 56.

Example: Find the unknown value in the proportion: 2 : x = 3 : 9.

$$2 : x = 3 : 9$$

First, convert the colon-based odds-notation ratios to fractional form:

$$\frac{2}{x} = \frac{3}{9}$$

Then solve the proportion:

$$\frac{2}{x} = \frac{3}{9}$$
$$9\,(2) = x\,(3)$$
$$18 = 3x$$
$$6 = x$$

Example: Find the unknown value in the proportion: (2x + 1) : 2 = (x + 2) : 5

$$(2x + 1) : 2 = (x + 2) : 5$$

First, convert the colon-based odds-notation ratios to fractional form:

$$\frac{2x+1}{2} = \frac{x+2}{5}$$

Then solve the proportion:

$$\begin{aligned} \frac{2x+1}{2} &= \frac{x+2}{5} \\ 5(2x+1) &= 2(x+2) \\ 10x+5 &= 2x+4 \\ 8x &= -1 \\ x &= -1/8 \end{aligned}$$

Example: If twelve inches correspond to 30.48 centimeters, how many centimeters are there in thirty inches?

Set up ratios with "inches" on top, and use "c" to stand for the number of centimeters

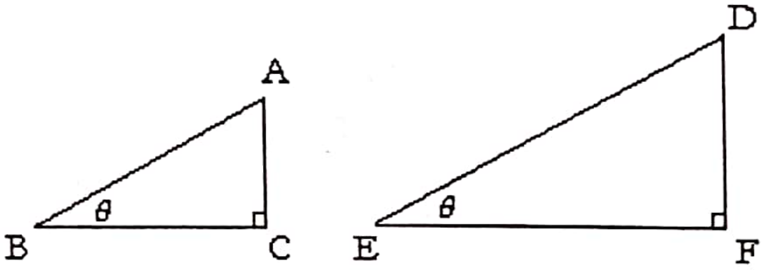
$$\frac{\text{inches}}{\text{centimeters}} : \frac{12}{30.48} = \frac{30}{c}$$

$$\begin{aligned} \frac{12}{30.48} &= \frac{30}{c} \\ 12c &= (30)(30.48) \\ 12c &= 914.4 \\ c &= 76.2 \end{aligned}$$

Thirty inches corresponds to 76.2 cm.

SIMILAR FIGURES

For triangles to be similar, however, it is sufficient that they be equiangular. Right triangles will be similar if an acute angle of one is equal to an acute angle of the other.



In the right triangles ABC, DEF, if the acute angle at B is equal to the acute angle at E, then those triangles will be similar. Therefore the sides that make the equal angles will be proportional. For example,

BC : CA = EF : FD.

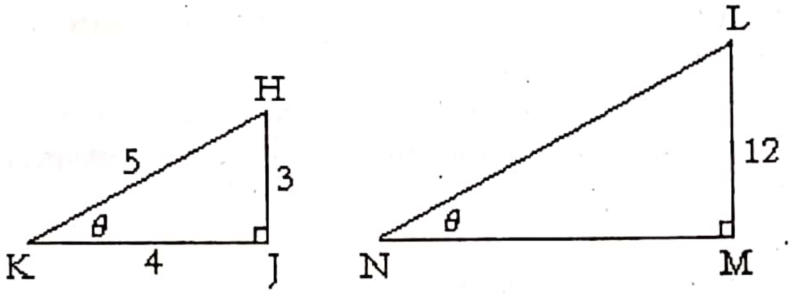
If BC were twice as long as CA, then EF would also be twice as long as FD.

And alternately, the corresponding sides are proportional:

BC : EF = CA : FD.

If BC were half of EF, then CA would also be half of FD.

Example: The right triangles HJK, LMN are similar, with HJ = 3 cm,



JK = 4 cm, and KH = 5 cm. If LM = 12 cm, then how long are MN and NL?

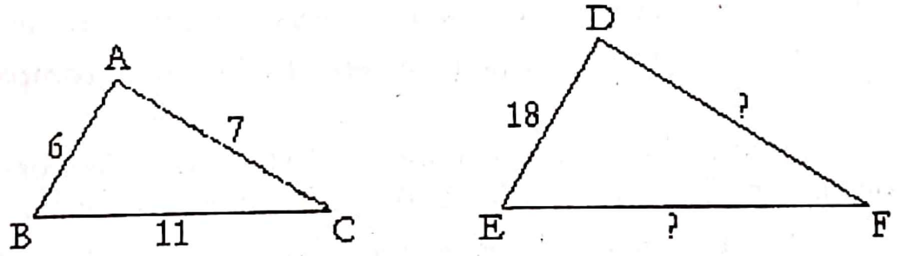
Alternately, LM : HJ = MN : JK.

Since LM is four times HJ, then MN will be four times JK -- it will be 16 cm.

Similarly, NL will be four times KH, it will be 20 cm.

To put the whole thing more simply: The side corresponding to HJ has been multiplied by 4. Therefore, since the triangles are similar, every side will be multiplied by 4.

Example: Triangles ABC, DEF are similar.



AB = 6 cm, BC = 11 cm, CA = 7 cm, and DE = 18 cm. How long are EF and DF?

The side corresponding to AB has been multiplied by 3. Therefore every side will be multiplied by 3. EF = 33 cm, and DF = 21 cm.

Example: The height of a eucalyptus tree in a park is 12 feet. It casts a shadow of 6 feet. A maple tree in the same park casts a shadow of 2 feet. Find the height of the maple tree.

Let x be the height of the maple tree.

x/2 = 12/6 [Write a proportion.]

6x = 12 × 2 [Write the cross products.]

6x/6 = 24/6 [Divide both sides by 6.]

x = 4 The height of the maple tree is 4 feet.

SELF-EXERCISE

1. Express 4/9 to 1/3 in its lowest term.
2. What is ratio between 3.5 and 7.5?
3. The cash price of Rs. 1000 has to be distributed b/w the two students in the ratio 2:3. How many does each student receive?
4. Bata Shoe Company has 50,000 pairs of shoes and management likes to supply these pairs of shoes to four whole sale dealers in the ratio of 4:7:9:5. How much pairs of shoes should each dealer receive?
5. Reduce to its lowest term, 24:64.
6. What is the value of the ratio 8 x 7: 9 x 13?
7. Three partners invested Rs. 18000, Rs. 16500 and Rs. 12500 respectively. If third partner got Rs. 4625, as a profit, what was the total profit? And Profit of 1st and 2nd partner?
8. Find the missing term from the proportion 2:7 :: ?:49.
9. If 20 men can prepare 10 office tables in a day. How many men are required to prepare 25 such office tables in a day?
10. If 15 dozen of eggs cost Rs. 202.50. How much will 8 dozen eggs cost?
11. If 20 men can construct a housing unit in 60 days. How many men are required to construct such housing unit in 48 days?
12. Rawal Express takes 4 and a half hour at a speed of 80 km / h to go from Rawalpindi to Lahore. How fast must the train travel to complete the same journey in 3 and a half hour?
13. Fifteen men complete the work in 20 days. How long it will take to complete the work by 10 workers?
14. A production manager plans to produce 100 units with the help of 25 workers who work 4 hours a day. How many units can be made by 40 workers if they work 3 hours / day?
15. If the stay of 12 men for 28 days in a hotel cost Rs. 6720, find the cost for stay of 8 men for 14 days in the hotel?

CHAPTER 24

GEOMETRY

Geometry is all about shapes and their properties. Geometry can be divided into: plane geometry and solid geometry. *Plane Geometry* is about flat shapes like lines, circles and triangles; the shapes that can be drawn on a piece of paper. *Solid Geometry* is about three dimensional objects like cubes, prisms, cylinders and spheres. Points, lines, and planes make up the basis of all geometry. Angles play an important role in creating architecture and they are critical in the study of geometry.

Common Symbols used in Geometry

Here are the most common geometrical symbols:

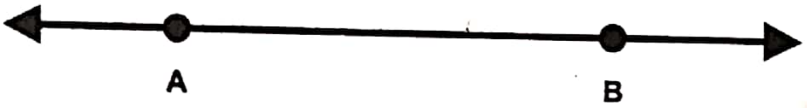
Symbol	Meaning	Example	In Words
Δ	Triangle	Δ ABC has three equal sides	Triangle ABC has three equal sides
∠	Angle	∠ABC is 45°	The angle formed by ABC is 45 degrees.
⊥	Perpendicular	AB ⊥ CD	The line AB is perpendicular to line CD.
‖	Parallel	EF||GH	The line EF is parallel to line GH.
o	Degrees	360°	360° makes a full circle
∟	Right Angle (90°)	∟ is 90°	A right angle is 90 degrees.
$\overline{AB}$	Line Segment "AB"	AB	The line between A and B.
$\overleftrightarrow{AB}$	Line "AB"	$\overrightarrow{AB}$	The infinite line that includes A and B.
$\overrightarrow{AB}$	Ray "AB"	$\overrightarrow{AB}$	The line that starts at A, goes through B and continues on.
≅	Congruent (same shape and size)	ΔABC ≅ Δ DEF	Triangle ABC is congruent to triangle DEF.

Definitions

Point: A point is a small dot. It has no length, width, or thickness.

Plane: A plane is a flat surface with no boundaries and no thickness.

Line: A line extends indefinitely in both directions indicated by arrows. A line can be identified by naming any two points on the line, for example, A and B. The notation for a line that extends through points A and B is $\overleftrightarrow{AB}$ (in alphabetic order).

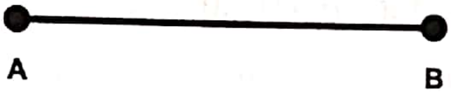


Parallel Lines: Lines that never intersect.

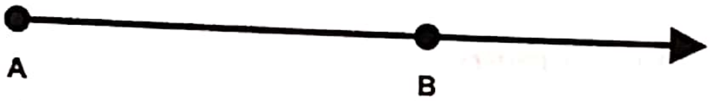
Intersecting Lines: Lines that have a point in common.

Perpendicular Lines: Lines that intersect at a 90° angle.

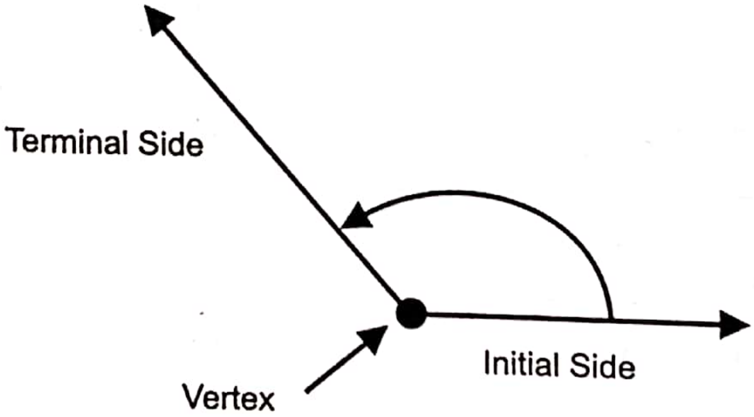
Line Segment: A line segments starts and stops at distinct points called endpoints. A line segment can be identified by its endpoints, for example, A and B. The notation for a line segment that includes the points A and B is $\overline{AB}$ (in alphabetic order).



Ray: A ray consists of a point on a line and all subsequent points on one side of the point. The point from which the ray originates is called the endpoint. A ray is named by its endpoint and any other point on the ray, for example, A and B. The notation for a ray that includes the points A and B is $\overrightarrow{AB}$ (in alphabetic order).



Angle: An angle is determined by rotating a ray about its endpoint. The starting position of the ray is the initial side of the angle, and the position after rotation is the terminal side of the angle. The point where the initial and the terminal side meet is called the vertex of the angle.

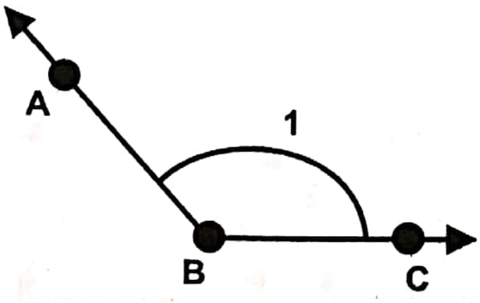


Note: When drawing an angle, the rotation of the ray about its endpoint is usually indicated with an arc in between the initial and terminal sides.

Most commonly, angles are measured in degrees. This is indicated by the degree sign ° next to a number. For example, 45°.

Naming Angles

Angles can be named in several ways. We'll show them by using the following picture:

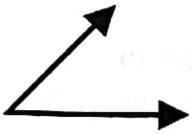


- We can place a number or letter in between the two rays, say 1 or the common Greek letter θ (theta) and then name the angle $\angle 1$ or using the symbol $\angle\theta$ for angle. We pronounce this "angle 1" or "angle theta."
- We can also use the letters of points on the rays together with the vertex point. That is, $\angle ABC$ or $\angle CBA$ either way, as long as the letter for the vertex point is in the middle.
- Finally, we can use the letter for the vertex point alone as long as it is perfectly clear which angle is designated by this letter. In the picture above, it is quite clear which angle we mean when we say $\angle B$.

Types of Angles

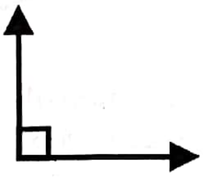
- **Right Angle:** Angles whose measure is exactly $90°$.
- **Straight Angle:** Angles whose measure is exactly $180°$.
- **Acute Angle:** Angles whose measure is greater than $0°$ but less than $90°$.
- **Obtuse Angle:** Angles whose measure is greater than $90°$ but less than $180°$.
- **Reflex Angle:** The reflex angle is the larger angle. It is more than 180° but less than 360°

Acute Angle



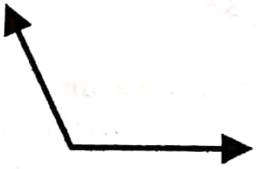
Less then 90°

Right Angle



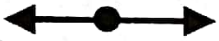
Exactly 90°

Obtuse Angle



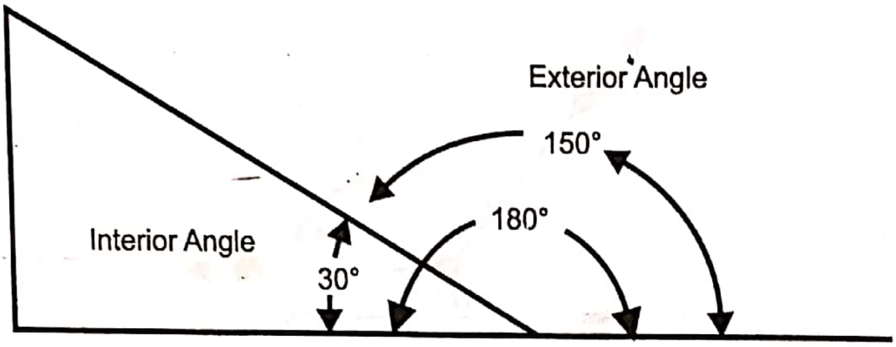
Between 90° and 180°

Straight Angle

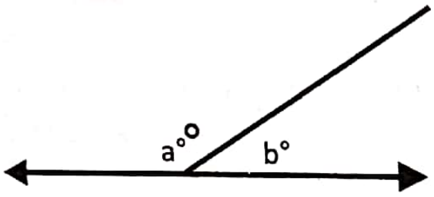


Exactly 180°

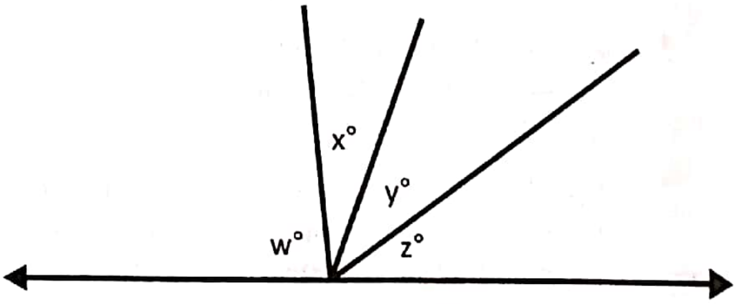
- **Complementary Angles:** Two angles are called complementary when their sum is 90°
- **Supplementary Angles:** Two angles are called supplementary when their sum is 180°
- **Interior and Exterior Angle:** An Interior Angle is an angle inside a shape. While the Exterior Angle is the angle between any side of a shape, and a line extended from the next side.



If two or more angles form a straight angle, the sum of their measures is 180°.



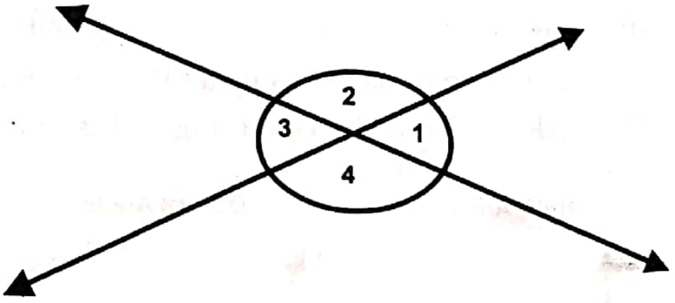
$a + b = 180$



$w + x + y + z = 180$

Rule 1: Relationships among angles Formed by Intersecting Lines

When two lines intersect, the opposite angles formed are called **vertical angles.**

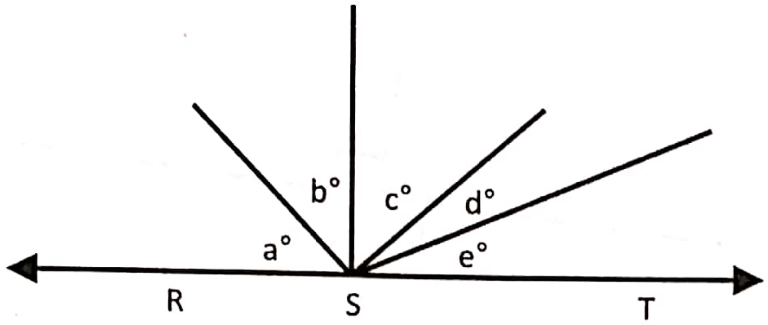


Note: When we discuss the **measure** of an angle, traditionally the letter **m** is placed before the angle symbol. We can say the following about the angles form by these lines: The measures of opposite or vertical angles are equal. In the picture above, $m\angle 1 = m\angle 3$. Likewise, $m\angle 2 = m\angle 4$ The sum of the measures of adjacent angles equal 180°.

In the picture above,

- $m\angle 1 + m\angle 2 = 180°$
- $m\angle 3 + m\angle 4 = 180°$
- $m\angle 1 + m\angle 4 = 180°$
- $m\angle 2 + m\angle 3 = 180°$

Example 1: In the figure below, *R*, *S*, and *T* are all on line *l*. What is the average of a, *b*, *c*, *d*, and e?



Solution: Since ∠ RST is a straight angle, the sum of a, b, c, d, and e is 180, and so their average is $\frac{180}{5} = 36$ **(b).**

Example 2: Classify the angles as right, straight, acute, or obtuse: $38°, 95°, 90°$ and $180°$

a. $38°$ - acute angle because the measure is greater than $0°$ but less than $90°$.

b. $95°$ - obtuse angle because the measure is greater than $90°$ but less than $180°$.

c. $90°$ - right angle

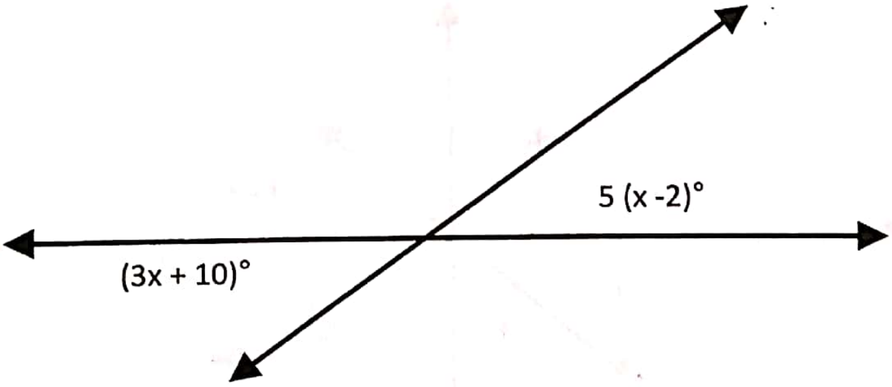
Example 3: Tell whether the angle pairs are complementary, supplementary, or neither: $(17°, 73°), (38°, 142°)$ and $(52°, 48°)$

a. $17°, 73°$ - sum equals $90°$, therefore, angles are complementary.

b. $38°$, $142°$ - sum equals $180°$, therefore, angles are supplementary.

c. $52°$, $48°$ - sum equals $100°$, which is neither complementary ($90°$) nor supplementary ($180°$).

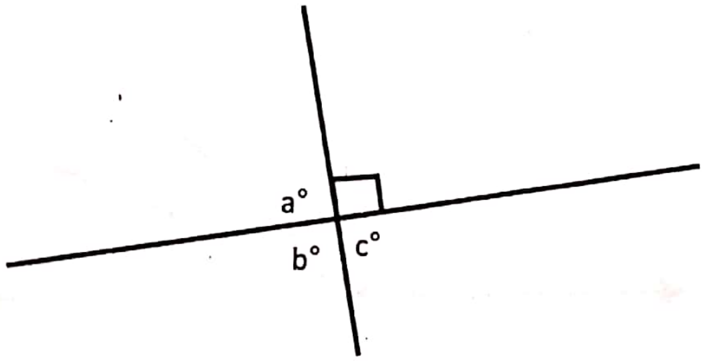
Example 4: In the figure below, what is the value of x?



Solution: Since the measures of vertical angles are equal,

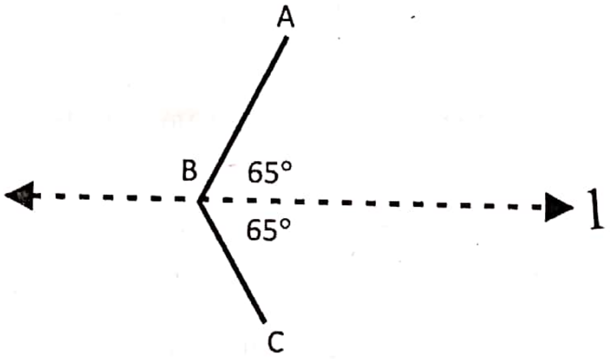
$3x + 10 = 5(x - 2) \Rightarrow 3x + 10 = 5x - 10 \Rightarrow 3x + 20 = 5x \Rightarrow 20 = 2x \Rightarrow$ $\mathbf{x = 10.}$

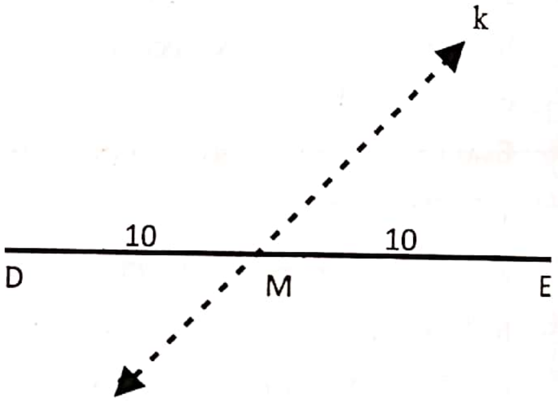
Rule 2: If one of the angles formed by the intersection of two lines (or line segments) is a right angle, then all four angles are right angles. Two lines that intersect to form right angles are called perpendicular.



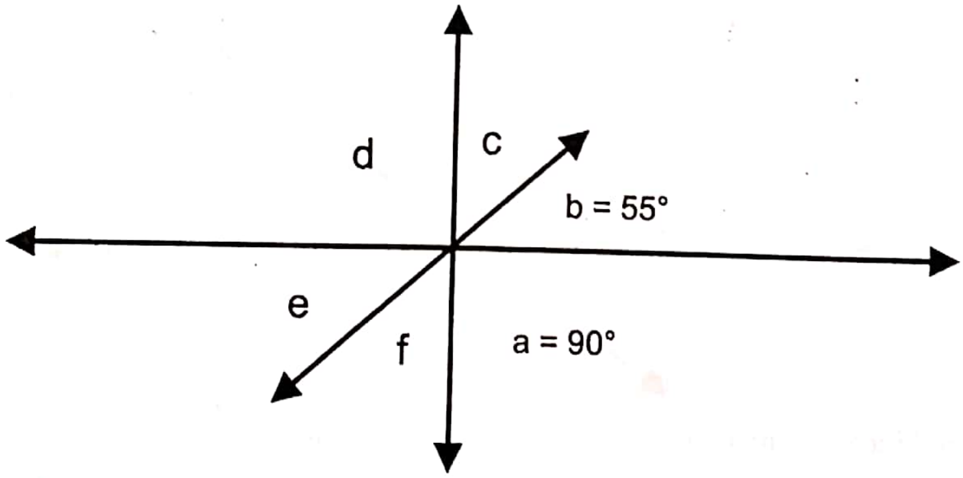
a = b = c = 90

In the figures below, line *l* divides ∠ *ABC* into two equal parts, and line *k* divides line segment DE into two equal parts. Line *l* is said to ***bisect the*** angle, and line k ***bisects the*** line segment. Point *M* is called the ***midpoint*** of segment *DE*.





Example 5: In the figure below, you are given the measure of two angles. Find the measure of the remaining angles.



Angles **a** and **d** are opposite angles. Therefore, m∠a equals the m∠d and this angle must be 90°.

Angles **b** and **c** are complementary angles. The measure of ∠c must be $90^\circ - 55^\circ = 35^\circ$.

Angles **b** and **e** are opposite angles. Therefore, measure of angle b equals m∠e. The measure of ∠e must be 55°.